Virtual Brands and Platform Intermediation*

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Abstract

Virtual brands, established by firms beyond their original brands to sell their existing products on online platforms, are gaining prominence on various food-delivery platforms. This paper studies a firm's decision to create multiple brands on an online platform and the platform's decision to recommend them to consumers who make their purchase decisions after searching recommended brands on the platform. We find a multi-product firm can utilize multiple identical-menu brands with different leading products to communicate information about its product variety, enticing more consumers to search its brands on the platform. Surprisingly, such information transmission by a multi-product firm raises not only consumer surplus but also the profit of the single-product firm that does not utilize virtual brands. We find that under privacy environment where the platform does not have access to consumer-preference information, it facilitates this information transmission by consistently recommending all brands to all consumers. By contrast, under no privacy where the platform knows consumer types, it uses brands with different leading products to target different consumer segments, which essentially restricts the information-transmission channel. Interestingly, however, the profits of both the platform and the virtual-brand-offering firm increase as a result. Finally, we show banning identical-menu virtual brands can further benefit both the multi-product firm and consumers when the ban pushes the multi-product firm to create multiple virtual brands specializing in distinct products. However, when the ban leads the multi-product firm to keep only one brand with all its products, the ban can hurt consumers and all firm types.

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1 Introduction

The online food-delivery industry is experiencing rapid growth, with projected global revenues reaching US$440 billion by 2024.[1] Traditionally, restaurants managed their deliveries directly; however, the industry has shifted predominantly towards transactions facilitated through a few major delivery platforms, such as UberEats, Grubhub, and DoorDash.[2] A few years ago, these platforms started allowing restaurants to offer multiple brands, with additional brands having only an online or "virtual" presence. For example, the Wingstop restaurant chain launched the virtual brand Thighstop alongside its original Wingstop brand on platforms, and Applebee's launched Cosmic Wings alongside Applebee's.[3] Figure 1 shows virtual brands offered by a local restaurant in Toronto on UberEats.

Notably, these virtual brands often offer identical menus of existing products under other brand names, which differs from the traditional umbrella branding strategy, in which firms introduce new products or target new consumer segments by leveraging the quality signals associated with the "umbrella" brand. In addition, such platform-driven markets feature algorithms that may significantly influence search results and recommendations, effectively determining the brands visible to each consumer segment. Furthermore, recently some platforms have begun regulating virtual brands. For instance, UberEats and DoorDash specifically restrict virtual brands with identical menus, requiring a certain percentage of unique items per brand.[4]

To investigate this novel marketing phenomenon and shed light on the role of virtual brands on online platforms, why a platform may regulate identical-menu virtual brands and how these choices affect all relevant stakeholders, we examine the interaction between the firm's creation of virtual brands and the platform's selective recommendation. The objectives of this paper are threefold: first, to find the benefits that firms accrue from creating virtual brands; second, to analyze the impacts of the virtual brands on consumers, firms that do not use virtual brands, and the platforms; and third, to explore the potential consequences of the platform allowing or banning virtual brands that use menus identical to the original brands.

We develop a model in which consumers buy products from a firm through a platform. There are two types of products and two types of consumers, each familiar with one product type and unfamiliar with the other. Familiarity is defined based on whether a consumer knows her utility for a product ex ante. A firm may be capable of producing only one or both of these two products and can create brands for selling them. In this paper, we focus on the online market mediated through the platform, so we do not distinguish virtual brands and the original brand. Instead, the technology of virtual brands simply make the creation of multiple brands possible and cost-less for the firm. The firm can decide on the number of brands to create, and for each brand, select a leading product that is visible to consumers prior to their search. Consumers conduct searches at

[1] https://www.statista.com/outlook/dmo/online-food-delivery/worldwide.

[2] https://www.mckinsey.com/industries/technology-media-and-telecommunications/our-insights/ordering-in-the-rapid-evolution-of-food-delivery.

[3] https://www.cnbc.com/2021/09/06/these-restaurant-chains-have-created-spinoff-virtual-brands-.html.

[4] https://www.wsj.com/articles/uber-eats-to-take-down-thousands-of-virtual-brands-to-declutter-the-app-2e05ca96; https://help.doordash.com/merchants/s/article/Official-DoorDash-Virtual-Brand-Quality-Requirements.

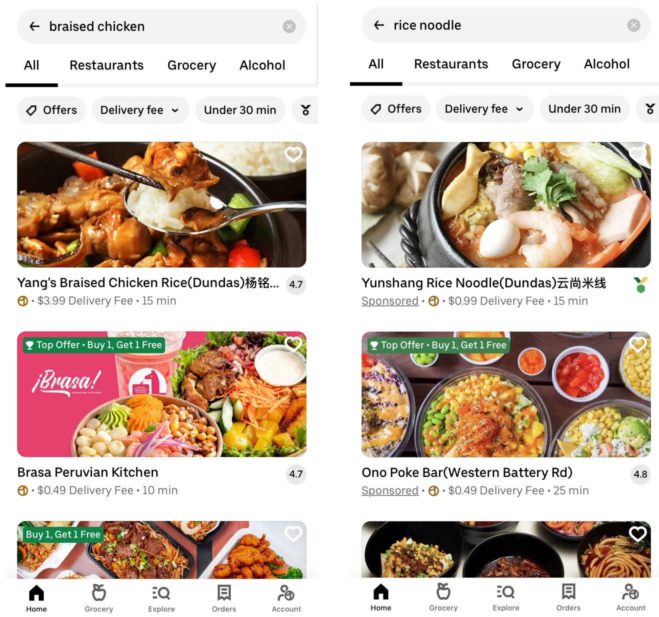

Figure 1: Virtual brands on Ubereats

Note: These are two search results on UberEats. The search on the left is for braised chicken, while the search on the right is for rice noodle. The first brand in each result is operated by the same offline restaurant, "Yunshang Rice Noodle," in Toronto.

the brand level, incurring a search cost for each brand. The platform, which collects a percentage fee proportional to firm revenue, assesses the consumers' preference types and presents a subset of the available brands to each consumer. The recommendation can be either type dependent or independent. Before initiating a search for any brand, consumers only take note of the brands recommended to them, as well as the leading product associated with each brand. Upon paying the search cost to visit a brand, they are able to observe the product variety, the prices of products within the brand, and the exact utility of the unfamiliar product, if it is offered by the brand.

We find that in the equilibrium the two-product firm launches two distinct brands, each offering both products but featuring a different leading product. By contrast, the single-product firm markets just one brand for its only product. Even though a two-product firm prices the same product identically across both brands, and consumers limit their search to a single brand, the mere existence of a second brand, with a different leading product, serves to communicate information on product variety, distinguishing the firm from its single-product counterparts.[5] Offering two distinct brands boosts the likelihood of consumers finding a desirable product, making them more inclined to visit such a brand. Concurrently, the firm can hike prices for both products, caring less about which specific product to sell. The added brand, seemingly redundant, actually carries informational value.

The platform plays a pivotal role in this information communication by showcasing all brands

[5]One could argue that the firm might communicate product variety by using different product images as the leading image or by including all product names in the brand name. However, due to limited screen space and other practical constraints, this approach is often not desirable.

to all consumers. In the privacy environment where the platform does not know or cannot make use of consumer-preference information to target consumers, the platform finds sustaining this informational channel is beneficial. It mitigates information asymmetry between consumers and the firm by consistently displaying all available brands to consumers, enabling the firm to use virtual brands to communicate product variety. Surprisingly, our findings indicate the advent of virtual-brand technology and the establishment of multiple brands by two-product firms contribute to an overall improvement in welfare. This enhancement extends beyond the two-product firm and the platform, positively impacting both consumers and single-product firms. Consumers benefit by making informed search decisions, intensifying their search efforts when the expected surplus is high and reducing them when the surplus is lower. The benefits accruing to the single-product firm as a result of its separation from the two-product firm are more nuanced. As a seller of only one product, the single-product firm is compelled to set a lower price to cater to the demand from both consumer types even conditional on consumer visit. As a result of not pooling with the two-product firm, the single-product firm's lower pricing strategy leads to increased consumer visits, proving advantageous for the firm. The platform would, by extension, benefit from this "win-win-win" equilibrium.

In comparison, in the no privacy environment where the platform has consumer-preference information and can use it to target consumers, the platform further benefits from making targeted recommendations. This finding highlights the second function of virtual brands: the multiple brands created by the two-product firm with distinct leading products enable the platform to target consumers. The platform would show only one of the brands to each consumer depending on their types. In this equilibrium, the firm loses its ability to use virtual brands to transmit product-variety information to consumers. Interestingly, the profits of both the platform and the virtual-brand-offering firm increase as a result. The targeted recommendation aided by the firm's creation of distinct brands makes the beliefs of heterogeneous consumers diverge prior to their search. Through targeted recommendation, the platform effectively guides consumers to intensify their search when the firm is better at making profits, that is, when the firm offers multiple products.

Our policy implication is that the ban of identical-menu virtual brands can elevate welfare for both firms and consumers. In scenarios where the utility of the familiar product is moderate, a two-product firm would establish two separate brands, each featuring a different product. Consumers self-select to visit the brand featuring their unfamiliar product initially and may opt to explore the other brand. The establishment of two distinct brands, each showcasing just one product, curtails the two-product firm's ability to exploit consumers, forcing the firm to lower prices as consumers self-select to visit the brand featuring their unfamiliar product first. Surprisingly, expectation of lower prices serves to attract more consumers to visit, resulting in a welfare improvement for both the two-product firm and consumers. However, achieving such a welfare-enhancing outcome necessitates the ban by the platform, because the two-product firm itself is otherwise consistently motivated to offer both products in its brand and elevate prices to maximize surplus extraction from consumers. The ban effectively limits the firm's ability to exploit consumers conditional on search, which in turn attracts more consumer traffic. Note that this equilibrium featuring two

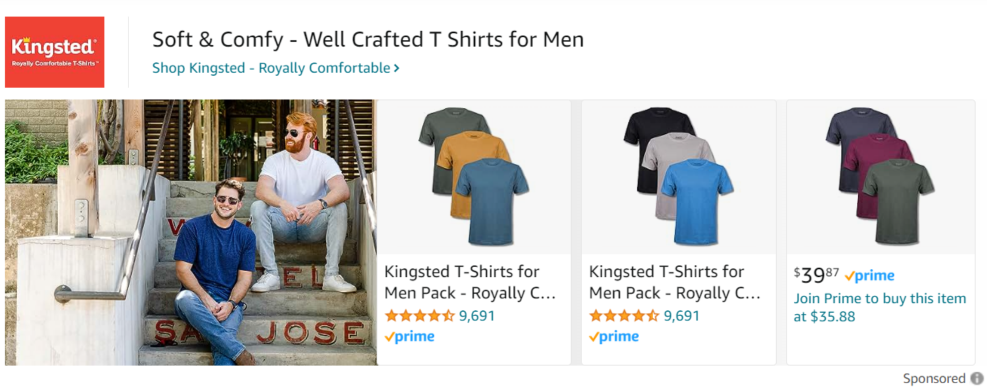

Figure 2: Multiple listings of the same group of products on Amazon

distinct brands is only feasible when the utility of the familiar product is not very high, ensuring consumers are indeed inclined to initiate their search with the brand featuring their unfamiliar product. Otherwise, the equilibrium scenario depicts the two-product firm opting to create a single brand offering both products, randomly assigning the leading product with an equal likelihood.

Similar marketing phenomena, though not explicitly labeled as "virtual brands," are prevalent in other industries. For example, on online marketplaces such as Amazon, firms often create multiple product listings offering the same group of products, but differentiated by color, design, or other minor variations (Figure 2). These multiple listings, akin to the multiple brands enabled by virtual brands, effectively communicate a firm's product variety to consumers. Such information can influence consumer search behavior and prompt firms to adapt their branding and pricing strategies accordingly. Thus, the insights we gain from the study of virtual brands can illuminate these analogous phenomena in other contexts.

1.1 Literature review

Our paper explores the novel marketing phenomenon of virtual brands and naturally contributes to the branding literature in marketing and economics. Traditionally, a brand is seen as a firm's identity tied to its products, and the literature often views branding as a way to convey vertical product information either through signaling or reputation concerns when consumers have incomplete information about product quality prior to consumption (Bronnenberg, Dubé, and Moorthy, 2019). Branding and brand advertising can serve as signals for product quality when there is a correlation between quality and advertising spending (Nelson, 1970; Kihlstrom and Riordan, 1984; Milgrom and Roberts, 1986). From a dynamic perspective, brands give high-quality-product manufacturers incentives to uphold their reputation through consistent branding (Klein and Leffler, 1981; Shapiro, 1983; Tadelis, 1999). Our paper adds to the literature by revealing a novel aspect of the informational value of branding. We show how multi-product firms can leverage virtual brands to disseminate information about their product variety, influencing consumer search behavior and the pricing strategies of all types of firms. Also on signaling about vertical information, a large body of literature focuses on umbrella branding and brand extension as means of conveying quality information when introducing a new product to a different market (Wernerfelt, 1988; Wernerfelt, 1991; Cabral, 2000; Moorthy, 2012; Yu, 2021). In both models on umbrella branding and our model, the ownership of the brands or the identity of firms are observable to consumers, which guarantees the effective communication of information to consumers. Our study, however, focuses on the same product market, and the two products are substitutes for consumers with unit demand in this market. Instead of quality information, our focus is on variety information transmitted through additional brands.

A few recent papers also study branding from various perspectives, for example, the impact of social factors such as match making and status goods in branding decisions (Kuksov, 2007; Amaldoss and Jain, 2015), strategic decisions of store brands versus national brands (Sayman, Hoch, and Raju, 2002; Scott Morton and Zettelmeyer, 2004; Soberman and Parker, 2006; Amaldoss and Shin, 2015), and brand positioning in horizontal differentiation models (Ke, Shin, and Yu, 2023). In an era where digital feedback and reviews are plentiful, thus eroding the traditional signaling value about quality (Bronnenberg, Dubé, and Moorthy, 2019), our paper posits that branding may carry informational value through another channel, such as variety information, that also impacts firms' pricing strategies. Several recent papers also analyze the activity of firms creating multiple brands from different angles and nicely complement our paper (Armstrong and Vickers, 2024; Grubb and Westphal, 2024). Their analyses focus on a single product, and consumers may not be aware of multiple brands selling the same product either through different exogenous consideration sets or through different search probabilities. They do not consider the role of the platform. By contrast, our paper highlights the effect of multiple products and the platform's selective recommendation by the platform. No consumer confusion exists about the brand ownership, and the awareness of brands depends on the endogenous decisions of the firm and the platform. In the online marketplace, if the concern is consumer confusion or market congestion, platforms could arguably manage brand visibility through their recommendation systems, potentially eliminating the need for additional branding restrictions.

Our research is related to the expansive literature on consumer search, starting from the seminal paper Diamond (1971). Wolinsky (1986) introduces sequential search with horizontal differentiation. In our model, consumers are heterogeneous in the product familiarity and utility of the unfamiliar products. They search sequentially among different brands created by a firm and filtered and presented by a platform. In Varian (1980), firms also face both informed and uninformed consumers, similar to our notion of familiar and unfamiliar product. However, their notion of familiarity is whether consumers know the price before search and the utility is fixed, whereas our notion is whether consumers know the utility of the product. Arbatskaya (2007) and Armstrong, Vickers, and Zhou (2009) study directed search. In the former, the entire search sequence is exogenously set, whereas in the latter, a prominent firm is exogenously selected to be searched first and the rest of the search sequence is random. By contrast, in our model, brands and their leading products are visible to consumers and consumers direct their search first to the brand that gives them highest expected consumer surplus. Our framework introduces uncertain product variety, leading to a potentially varying proportion of consumer visits to each brand. Guo and Wu (2016) explores consumer search decisions under uncertainty about product quality, with price acting as a quality signal before the search. Our model, by contrast, introduce uncertainty about product variety, the exact utility of unfamiliar products and product prices for consumers, whereas the brands and the leading product of each brand are observable to consumers and may serve as signals through filtering by the platform.

Our paper also contributes to the recent literature on search design, information transmission, and recommendation strategies within the platform economy. Hagiu and Jullien (2011) explores a scenario where a platform controls the search sequence for heterogeneous consumers between two single-product firms. By contrast, our paper positions the platform to selectively recommend a list of brands, whereas consumers retain their control over their search sequence among these brands. Dukes and Liu (2016) considers the design of the online marketplace hosting multiple sellers with ex-ante homogeneous product, and the platform can directly alter the search cost for consumers. In our paper, the search cost is exogenous to the platform and the design is the recommendation list. Jiang and Zou (2020) delve into a platform's optimal setting of transaction fees and assess the impacts of lowered search costs and filtering based on observable attributes. In Ke, Lin, and Lu (2022), the platform directly uses information design to change consumers' belief about their matched product before they search and firms bid to win a prominent position. Zhong (2023) investigates a threshold model for product filtering based on match value and price, examining how a platform balances these factors depending on precision. In comparison to these papers, in our model, the percentage fee is fixed and our research is centered on selective recommendations by the platform and restrictions imposed on virtual brands. Additionally, Zhou and Zou (2023) analyze how personalized product recommendations by online markets influence seller competition, considering varying degrees of consumer profiling accuracy. Their model features a platform recommending a single product to consumers, who may or may not be aware of other available products outside the recommendation. Our study sets itself apart by presenting a scenario where the platform recommends a list of brands that are directly observable to consumers, along with leading products of each brand. However, discovering prices and the variety within each brand involves a search process that incurs a cost.

2 Model

Consider a firm that sells an uncertain variety of products in a market consisting of a unit mass of consumers. For simplicity, suppose there are only two possible products, $i \in \{1, 2\}$. The firm may be capable of producing one or both products. With probability γ, the firm can produce only one of the two products (either product 1 or 2). We refer to such a firm as a single-product firm. If a single-product firm can produce only product 1, we refer to it as a j_1 firm, labeled as $t = j_1$. Similarly, a firm that can produce only product 2 is referred to as a j_2 firm and is labeled as $t = j_2$. A single-product firm can be a j_1 firm or a j_2 firm with equal probability. With probability $1 - \gamma$, the firm can produce both products, and we refer to such a firm as a two-product firm, labeled as $t = j_{12}$. The firm's product variety is its private information. The marginal cost of production is zero for both the products.

The firm sells products to consumers using an independent online platform. When a consumer arrives at the platform to buy a product, the platform presents a list of brands to her. Each brand consists of a menu of products and their prices. However, the list of brands presented to the consumer displays only a product image (which is referred to as the image of the leading product) for each brand in the list. The information about all the products offered within the brand and their prices become known to the consumer only when she searches that specific brand by clicking at it. A key feature of our model is that consumers search sequentially at the brand level (and not at the firm or product level). To reach potential consumers, the firm establishes brands on the platform. When virtual brands are not available, each firm can just create one brand; when virtual brands are available, each firm can create multiple brands. The firm decides the number of brands, the product menu offered within each brand (subject to its own product variety), the price for each product within each brand, and the leading product for each brand.[6] We represent the price of a single-product leading-i brand for its sole product i by p_{s_i} and prices for a two-product brand by a pair (p_1, p_2), where p_i is the price of product i. We assume the cost of introducing a new brand is infinitesimally small, implying the firm will establish a brand only if doing so yields strictly higher profits.

Consumers are potentially interested in these two products and have unit demand, but they vary in their preferences between the two. Specifically, there are two consumer types, $\theta \in \{L, R\}$, present in equal proportions in the population. A type-L consumer is familiar only with product 1, whereas a type-R consumer is familiar only with product 2.[7] A consumer's valuation v for her familiar product is deterministic. However, a consumer's valuation u for her unfamiliar product is stochastic, and $u \sim U[0,1]$ is independent across consumers. We assume $v > 1/2$. That is, consumers have a higher expected value for their familiar product than their unfamiliar product. We consider the opposite case where the familiar product offers a lower expected value, $v < 1/2$, in Section 5.2. A consumer's product valuations are specific to products and do not depend on the brands they are offered in. Although v is publicly known by all players ex ante, the realization of u is learned only privately by each consumer after they search a brand containing their unfamiliar product. A consumer buys at most one unit of the product.

We consider two information environments: a privacy environment where each consumer's preference type θ is privately observed by the consumer herself, and a no privacy environment where each consumer's preference type θ is observed by both the consumer and the platform. The platform's information on consumer type may come from consumers' keyword search or behavioral tracking of past activities. The platform displays a non-empty subset of available brands to each consumer. Consumers learn about only those brands the platform displays to them. Consumers observe the leading product i of each displayed brand and form a belief β^i about the brand being a single-product brand. Note β^i is an endogenous variable and is derived using Bayes' rule wherever

[6] The main model focuses on a monopoly firm. In Section 5.1, we consider how virtual brands affect equilibrium with competitive firms.

[7] Introducing some degree of unfamiliarity helps to circumvent the Diamond paradox. The Diamond paradox says that when consumers share homogeneous valuations and have positive search costs, the unique equilibrium of the sequential search is that the firm sets the monopoly price and consumers do not search at all.

possible.

Subsequently, a consumer can incur a search cost $c \sim U[0, \bar{c}]$ (where $\bar{c} \geq 1$) per brand to learn the menu of product/s (and their price/s) offered within that brand.[8] Consumers try to find their best purchase option by searching sequentially. The search cost c is individually and independently drawn and independent of consumer preference type θ.[9] The search cost c is privately observed by each consumer before search. Upon searching a brand, consumers discern the exact utility of each product offered in that brand. Each consumer decides whether and which brand to search first, whether to search a second brand, and whether to buy a product or leave the market without any purchase after each search. The number of brands and the leading product of each brand are observable to consumers before they search. Therefore, they may affect consumers' sequential search decision. The product variety—other than the leading product—and product prices are unobserved to consumers before they search. Therefore, they can affect consumers' decisions only through consumers' expectations.

The platform levies a transaction fee on the firm (for using the platform to sell its products) that extracts a proportion of the firm's revenue (or, equivalently, its profit under the current model setup). Therefore, from the platform's perspective, maximizing its own profit is equivalent to maximizing the sum of expected profits of all firm types.[10] The platform decides which brands to recommend (or display when they arrive at the platform) to consumers. The platform can offer either a non-targeted or targeted recommendation. A recommendation is non-targeted if the platform displays the same subset of brands regardless of the consumer's familiar product. In the case of a targeted recommendation, the platform recommends different subsets of brands to different types of consumers. Note that in our model, we assume the platform only presents an unordered subset of brands to each consumer so that the order of brands within the subset does not convey any information.[11] The search order within a list of brands is decided entirely by consumers instead of the platform, which is different from the literature on prominent search (Armstrong, Vickers, and Zhou, 2009; Hagiu and Jullien, 2011).

The platform's recommendation strategy falls into one of three categories: (i) recommending all available brands to all consumers (full recommendation), (ii) recommending the same subset of brands to all consumers (non-targeted selective recommendation), or (iii) recommending different subsets of brands to consumers based on their familiarity with specific products (targeted recommendation). Only the first strategy reveals all available brands to consumers; the other two strategies filter brand information, for example, by showing only one brand when multiple brands

[8]The assumption $\bar{c} \geq 1$ ensures some consumers who decide not to search exist in the market. This assumption also simplifies the analysis.

[9]We model the privacy environment based solely on preference type θ rather than search cost c. This focus is due, first, to the greater importance of preference type for the firm, and second, to the fact that preference information can be more easily learned over time, whereas search cost is more variable.

[10]In the model presented in Section 5.3, we study an alternative setup where the platform receives a fixed per-transaction fee. As a result, the platform's objective in that setup is to maximize total transaction volume.

[11]If the platform can present a ordered list, it can also target consumers with the same subset but different orders of brands. As it turns out, with consumers ultimately deciding the search order, targeting using the same subset but different orders of brands is not effective and does not change the equilibrium outcome. Therefore, for ease of exposition, we assume the subset of brands is unordered.

exist on the platform. While the second category may seem redundant if the firm does not create multiple brands, including it ensures a complete consideration of the platform's strategy set and highlights the potential value of the platform's ability to commit to a specific selection of brands. The firm, however, cannot commit to a predetermined probability mix for creating different brands. The platform selects the recommendation strategy that maximizes its expected profit. In a privacy environment, the platform can choose only from non-targeted recommendation strategies, whereas in a no-privacy environment, all three strategy categories are available.

The timing of decisions is as follows. First, the platform commits to a strategy for recommending brands to consumers.[12] Next, the firm decides the number of brands, the product variety and the leading product for each brand, and the prices of all products within each brand. The platform then presents a list of brands to consumers according to its recommendation strategy. Consumers observe the list of brands recommended to them on the platform and form expectations about product variety and product prices. Subsequently, consumers decide the order of search. Then, they sequentially search the list of brands until deciding to buy a product or opting out. Finally, payoffs are realized.

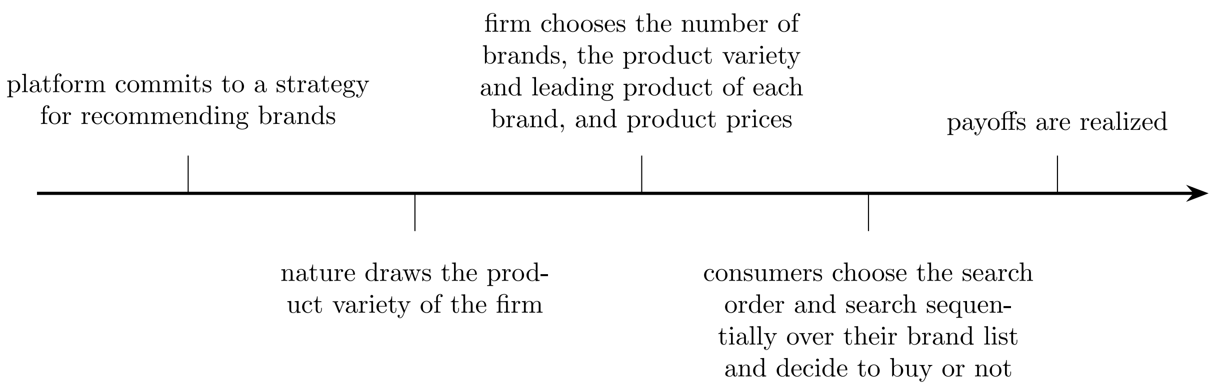

Figure 3: Timing of the game

We look for the Perfect Bayesian Equilibrium (PBE) of the game and adopt intuitive criterion. The equilibrium consists of a specification of the platform's recommendation strategy, the firm's branding and pricing strategies, consumers' search strategies, and their beliefs about product variety and product prices for each brand before they search. Consumers would form an expectation of the product variety offered by each brand, based on the brands recommended to them and their leading products, which is the result of both the firm and platform's strategy, and form an expectation of the product prices. When consumers are indifferent among several options, we assume they will choose each of these options with equal probability. Here, the brands and their leading products may serve as an information-communication device for the firm. If consumers were to observe an off-equilibrium action from any firm type, they would only ascribe it to firm types that may benefit from the deviation.

[12]Because the platform and the two-product firm often share a common interest, the platform's ability to commit to a recommendation strategy and its effect on the creation of multiple brands is straightforward.

3 Equilibrium Analysis and Results

In this section, we present the equilibrium analysis and generate insights about the firm's and the platform's decisions. We start with the analysis of consumer and firm strategies that can possibly be chosen in an equilibrium. Subsequently, in Section 3.2, we assume the platform recommends all available brands to consumers and solve the candidate equilibrium in which the two-product firm creates two brands. Section 3.3 analyzes the candidate equilibrium in which the two-product firm creates only one brand. Section 3.4 presents insights about the effects of creating virtual brands. Finally, Section 3.5 presents the analysis and insights about the platform's strategies under privacy (that is, when the platform cannot use consumer preference information) and no-privacy settings.

3.1 Consumer and firm strategies

A consumer is presented with a list of brands and observes each brand's leading product. Based on this limited information, she forms expectations about product variety and prices, calculating her expected surplus from visiting each brand. She will first visit the brand with the highest expected surplus, provided her search cost is lower than the expected surplus. Upon visiting, she may either purchase a product from this brand, leave the market without buying, or pay an additional search cost, c, to visit another brand, if available.

For instance, consider the scenario in which a type-L consumer only sees one leading-1 brand, holds belief β^1 of it providing just product 1, and expects the prices to be p_{s_1} for a single-product brand and (p_1, p_2) for a two-product brand where p_i is the price of product i. In this case, the expected consumer surplus for a type-L consumer visiting the single-product brand is simply

$$\begin{cases} v - p_{s_1} & \text{if } p_{s_1} \leq v, \\ 0 & \text{if } p_{s_1} > v, \end{cases}$$

whereas the expected consumer surplus visiting a two-product brand is

$$\begin{cases} \int_0^{v-p_1+p_2} (v-p_1)du + \int_{v-p_1+p_2}^1 (u-p_2)du = \frac{(1-p_2)^2}{2} + \frac{(v-p_1)^2}{2} + p_2(v-p_1) & \text{if } p_1 \leq v, \\ \int_{p_2}^1 (u-p_2)du = \frac{(1-p_2)^2}{2} & \text{if } p_1 > v. \end{cases}$$

A type-L consumer calculates her expected payoff from visiting this leading-1 brand as a weighted sum of the single-product brand with probability β^1 and the two-product brand with probability $1 - \beta^1$ and similarly for other cases with different sets of available brands. She then ranks them in descending order of expected consumer surplus and searches sequentially. She would buy if the realized payoff is positive and higher than the expected consumer surplus from the next brand minus search cost c, continue search if the expected consumer surplus from the next brand minus c is positive and higher than the current realized payoff, and leave the market without buying if both payoffs are negative.

Now, we can refine the set of strategies the firm may choose in equilibrium. The choices of product variety and product prices are unobserved to consumers before they search and hence do not affect consumer search decisions directly. The firm's revenue comes from both types of consumers, and the two types of consumers may have different probabilities of visiting a certain brand. For visits to each brand, let α denote the proportion of type-L consumers (with $1-\alpha$ representing type-R consumers). α is an endogenous variable determined by consumers' search decisions. For simplicity, we use a single α here, although it may vary across brands and scenarios for different equilibria. First, the firm would only set prices equal to or below v as long as the proportion of two types of consumers are not too lopsided.[13] If a product price exceeds v, only unfamiliar consumers might purchase. As $v > 1/2$, reducing price below v would attract some familiar consumers to buy and also increase the profit from unfamiliar consumers. Therefore, the prices will be weakly below v.

Next, for product variety, a single-product firm would create a single brand featuring its sole product. Because the single-product firm can only create brands with its only product, if it creates multiple brands, they would be identical from consumers' perspective. Consumers would not gain from searching more than one of them. If the single-product firm uses duplicate brands as an informational channel to attract more consumers, the two-product firm can easily mimic it, so the benefit of duplicate brands cannot be sustained. Therefore, with an arbitrarily small cost of creating a brand, the single-product firm creates only one brand. The two-product firm can establish up to four brands, each potentially offering one or two products, with one of them being the leading product. Two brands with the same two products but different leading products are distinct brands.[14] We show in Lemma 1 that the two-product firm will offer both products for any brand it creates, and as a result, it will create only one or two brands in equilibrium.

Lemma 1. *The two-product firm will offer both products for every brand it creates.*

The product variety is revealed to consumers after they search. For the two-product firm, by offering both products in a brand, consumers are more likely to find a good match for their heterogeneous preferences and buy one of the two products. In addition, the two-product firm can raise prices to extract more profits, because it does not care about which product is sold to consumers. On the other hand, the two-product firm cannot credibly commit to not providing both products before consumers search. In fact, in most cases, the optimal prices are raised to v so that consumers can either buy their unfamiliar product in case of a high value or be guaranteed to buy their familiar product at their reservation value. We take the optimal prices to be (v, v) for the two-product firm to illustrate our results, and in the proofs, we discuss the extreme cases that do not arise in equilibrium. As a result, regardless of the proportion α of type-L consumers, the two-product firm earns profit v conditional on consumer visit and leaves any consumer with

[13] The exception for the two-product firm is discussed in the proof of Lemma 1.

[14] Again, we can easily see duplicate brands would not exist in equilibrium. The reason is that consumers decide whether and which brand to search, without knowing the product variety in each brand other than the leading product. If a two-product firm were to create two identical brands, a single-product firm would then have an incentive to mimic if it attracts more consumers to search. If not, the two-product firm has an incentive to mimic this single-product firm. In either case, creating duplicate brands does not bring any value and cannot serve as a communication device either, so it cannot be an equilibrium play for the firm.

surplus

$$\int_v^1 (u-v)du + v \cdot 0 = \frac{(1-v)^2}{2}.$$

For j_1 firm, given the proportion of type-L consumers being α, the expected profit of setting price p conditional on consumer visit is $\alpha p + (1-\alpha)p(1-p)$, with optimal price given by

$$p(\alpha) = \begin{cases} \frac{1}{2} + \frac{\alpha}{2(1-\alpha)} & \text{if } \alpha \leq 1 - \frac{1}{2v}, \\ v & \text{if } \alpha > 1 - \frac{1}{2v}. \end{cases}$$

The unconditional expected profit of the firm is the conditional expected profit multiplied by a constant that is not affected by the choice of p directly but depends on consumers' expectation, similarly for j_2 firm.

In the following analysis, we begin by analyzing the subgame equilibrium assuming the platform recommends all available brands and then show the platform would indeed do so in the privacy equilibrium. Given Lemma 1, the two-product firm would just establish brands with both products. The next decision for the firm is to choose the leading product for each brand. As we argued, creating two identical two-product brands with the same leading product won't be an equilibrium. Therefore, we focus on two possible equilibrium types: a two-brand (separating) equilibrium where the two-product firm establishes two brands with distinct leading products, and a single-brand (pooling) equilibrium where the two-product firm establishes just one brand, equally likely to be leading-1 and leading-2. The single-brand equilibrium we choose is the symmetric one among all possible single-brand equilibria, and we show in the proof of Lemma 2 that it's the only possible one if each firm can only create one brand.

The two-brand equilibrium is the unique equilibrium when the platform recommends all available brands to each consumer (full recommendation). Hereafter, we may refer to the two-brand equilibrium as the equilibrium.We summarize this result in the proposition here and describe these two equilibrium types in detail in the following subsections. Some of the proofs regarding these equilibrium types are left to the appendix.

Proposition 1. *If the platform chooses full recommendation, in equilibrium, the two-product firm would create two brands with distinct leading products to convey it offers both products in both brands and set prices v for both products in its brands.*

As a single-product firm can only create brands with its sole product, the two-product firm can "choose" whether to separate itself from a single-product firm by creating both leading-1 and leading-2 brands or to pool with a single-product firm by creating just one brand. Because consumers are heterogeneous in their preferences, having one brand with only one leading product observable to consumers may deter some consumers from search. The two-product firm, by convincing consumers it offers both products, can increase average consumer visits with a wider expected range of choices. Therefore, the two-product firm would choose to set up two brands with distinct leading products to convey such information.

3.2 Two-brand equilibrium

In the two-brand equilibrium, three firm types create different portfolios of brands. Naturally, the j_1 firm creates a leading-1 brand and the j_2 firm creates a leading-2 brand, whereas the j_{12} firm creates both brands. Consumers observe the set of brands and rationally expect the product variety a firm can produce and offer in each brand. When both leading-1 and leading-2 brands are available, they rationally expect both products to be available in both brands and select the one with the more favorable expected prices. As discussed above, the j_{12} firm sets $p_1 = p_2 = v$ for products in both brands to maximize its profit. In anticipation, consumers would randomly explore one of the brands. The expected consumer surplus from a two-product brand before visiting is simply $(1-v)^2/2$, which is the same for both consumer types. As a result, an equal proportion of consumers visit each brand. The j_{12} firm's unconditional profit is $v(1-v)^2/(2\bar{c})$. Here, creating two brands does not bring additional consumer visits directly, because consumers would still choose just one of the brands to visit. However, the presence of two distinct brands shows consumers the brand is created by a two-product firm. Combined with the result that the two-product firm offers both products in each brand, consumers' expectation of the product variety also changes, which makes them more likely to find a match and hence more likely to visit on average.

A single-product firm can only establish brands with its single product. Upon observing just one brand, consumers are certain it offers only that specific product prior to their visit. The firm would create one such brand and set the price $p^* \leq v$ to maximize profit upon consumer visit. Take the j_1 firm as an example. The expected surplus from visiting its brand for type-L consumers is $v - p^*$, whereas for type-R consumers, the expected surplus is $(1-p^*)^2/2$. Consumers with search costs below these expected surpluses will visit the brand, so the proportion of type-L consumers is $\alpha_{j_1} = (v-p^*)/(v-p^*+(1-p^*)^2/2)$. The firm's expected profit of setting price p is

$$\frac{1}{2}\frac{(1-p^*)^2}{2\bar{c}}p(1-p) + \frac{1}{2}\frac{v-p^*}{\bar{c}}p.$$

The firm's optimal product price, p^*, satisfies

$$p^* = \frac{1}{2} + \frac{v-p^*}{(1-p^*)^2} \in (\frac{1}{2}, v). \tag{1}$$

Here we use the condition that the equilibrium optimal price the firm sets is consistent with consumers' expectation of it. The j_1 firm balances the profits from two types of consumers. The optimal price facing only type-L consumers is v, and the optimal price facing only type-R consumers is $1/2$, so the optimal price serving both types lies between these two prices. In the proof of Proposition 1, we show the j_{12} firm has no incentive to deviate by pretending to be a single-product firm.

3.3 Single-brand equilibrium

If the two-product firm just creates one brand, different firm types will be pooled together. The candidate single-brand equilibrium we consider here is a special case of more general single-brand candidate equilibria. More specifically, if the technology of virtual brands is not available or if the firm is restricted to hosting one brand only, this pooling case, as we show, is the unique equilibrium. For this single-brand equilibrium, the two-product firm chooses the one brand it creates to be a leading-1 or a leading-2 brand with equal probability. For each leading-i brand, consumers are uncertain whether it provides just product i or both before they visit, and they assign probability $\beta^i = \gamma$ to it being a single-product brand. Again based on previous analysis, the two-product firm would charge prices $p_1 = p_2 = v$. A single-product firm would just create one brand with its only product and optimally charge price p^P. Take leading-1 brand and j_1 firm as an example. Type-L consumers' expected surplus of visiting a leading-1 brand is

$$\gamma(v - p^P) + (1-\gamma)\frac{(1-v)^2}{2},$$

whereas type-R consumers' expected surplus of visiting a leading-1 brand is

$$\gamma\frac{(1-p^P)^2}{2} + (1-\gamma)\frac{(1-v)^2}{2}.$$

The j_1 firm maximizes profits from two types of consumers by choosing p^P that satisfies

$$p^P = \begin{cases} \frac{1}{2} + \frac{\gamma(v-p^P)+(1-\gamma)(1-v)^2/2}{\gamma(1-p^P)^2+(1-\gamma)(1-v)^2} \in (p^*, v), & \text{if } \gamma > 2(1-v), \\ v & \text{if } \gamma \leq 2(1-v). \end{cases} \tag{2}$$

When the probability of the single-product firm γ decreases, the single-product firm will set a higher price. If product prices were observable before consumers search, the reasoning would be that consumers' decision to visit a leading-1 brand depends more on the pricing of the two-product firm, so a higher price by the j_1 firm can increase the profit margin and reduce demand only slightly. However, as consumers do not observe product prices before they search, prices should not affect consumer decisions directly. Instead, the increasing dependence of consumer surplus on the two-product firm favors type-L consumers more than type-R consumers for visiting a leading-1 brand, so disproportionally more type-L consumers would visit the leading-1 brand when γ decreases. The j_1 firm would therefore lean more weight on profit from type-L consumers, and charge a higher price. When γ is small enough, the j_1 firm just charges v for product 1, the optimal price facing only type-L consumers.

In the proof of Proposition 1, we show the single-brand equilibrium does not exist with virtual brands available, because the two-product firm will deviate to create two distinct brands to credibly communicate to consumers their product variety and attract more consumers on average. Regarding the argument that the j_{12} firm cannot create two identical brands with the same leading products to signal that they produce both products, either the j_1 or j_2 firm will simply mimic by creating two of the same brands in order to attract more consumers.

Although the symmetric single-brand candidate equilibrium does not exist when creating virtual brands is possible, it serves as a benchmark equilibrium. That is, it is the unique single-brand equilibrium.

Lemma 2. *If limited to one brand for each firm, the two-product firm would create one brand with both products and choose the leading product with equal probability.*

If the two-product firm chooses the leading product with asymmetric probability, say, product 1 with probability ξ and product 2 with probability $1-\xi$, leading-1 and leading-2 brands would attract different composition and different amount of consumer traffic. For instance, if $\xi > 1/2$, the leading-2 brand will attract consumer visits and the two-product firm would deviate to always choose the leading-1 brand and set $\xi = 0$, so it cannot be an equilibrium.

3.4 Firm profits and consumer welfare

In this subsection, we compare the firm profits and consumer welfare in two ways. The first is to compare the single-product firm and the two-product firm in equilibrium to see the effects of wider product variety. The second is to compare the two-brand equilibrium with the single-brand equilibrium to see the effects of having virtual brands.

For the first comparison, in equilibrium, compared with consumer surplus provided by the j_1 firm, the expected surplus provided by the j_{12} firm for type-R consumers is smaller:

$$\frac{(1-v)^2}{2} < \frac{(1-p^*)^2}{2},$$

whereas for type-L consumers, the j_{12} firm offers a higher surplus:[15]

$$\frac{(1-v)^2}{2} > v - p^* \iff p^* > v - \frac{(1-v)^2}{2}.$$

Therefore, type-L consumers derive a lower expected surplus from visiting a leading-1 brand by the j_1 firm than a leading-1 brand by the j_{12} firm, vice versa for type-R consumers.

Corollary 1. *In equilibrium, for visits to a leading-i brand, relative to a single-product firm offering it, the two-product firm offering it provides more consumer surplus for consumers unfamiliar with product i, and less surplus for consumers familiar with product i,* $i \in \{1, 2\}$.

The j_1 firm faces demand from both types of consumers, and can extract surplus more effectively from type-L consumers with deterministic preferences. This benefits type-R consumers with

[15] Given that the optimal price p^* satisfies

$$p^*(1-p^*)^2 - \frac{1}{2}(1-p^*)^2 - (v-p^*) = 0,$$

and considering the first order derivative of the left-hand side, we can show the optimal price p^* exceeds $v - \frac{(1-v)^2}{2}$.

uncertain valuations. On the other hand, the j_{12} firm is more flexible because it provides both products to consumers and more effective at extracting surplus from type-R consumers.

The second comparison is between the welfare results under the two-brand equilibrium and those under the single-brand equilibrium.

Proposition 2. *Having virtual brands available, the two-product firm increases its own profit, and also benefits both types of consumers and the single-product firm.*

Consumers decide whether to visit a brand before they observe what is provided and the product prices except for the identity of the leading product. Without virtual brands, the two-product firm has to choose one of the products to be the leading product and be pooled with a single-product brand. Virtual brands enable the two-product firm to reduce information asymmetry by communicating to consumers before they search that it provides both products. Consumers would hence know both products are provided when they see two brands with distinct leading products and randomly choose one to visit. Because consumers have heterogeneous preferences, such information makes "correct" consumers–those who are more likely to buy–more likely to visit. Therefore, balancing consumer traffic from both types, a two-product brand can attract more consumer visits on average and the firm can extract more surplus from them. Interestingly, consumers and the single-product firm also benefit from it. On the one hand, before consumers search a brand, they know whether it offers both products or a single product. Consumers would search more if they expected to find a desirable product at a reasonable price. The variety information helps them search more intensely when they expect a higher surplus and less intensely when they expect a lower surplus. On the other hand, the single-product firm charges a lower price because it faces more uneven demand–disproportionally more unfamiliar consumers would search its brand. Anticipating this change in demand, the single-product firm would set a lower price, which increases consumer welfare, thus benefiting the single-product firm itself with higher demand. Virtual brands not only make the pie larger, but also benefit all the players.

3.5 Platform's recommendation

Given that the two-brand equilibrium is the unique subgame equilibrium when the platform recommends all available brands to each consumer, we can examine whether doing so is the platform's best interest. As the platform collects a fixed proportion of the firm's revenue (profit), we can directly compare the industry profit across all firm types. From the preceding results, the industry profit when the platform reveals all available brands is

$$\gamma \frac{{p^*}^2(1-p^*)^2}{4\bar{c}} + (1-\gamma)\frac{v(1-v)^2}{2\bar{c}}. \tag{3}$$

As the single-product firm just creates one brand, the selective recommendation of the platform occurs for the two-product firm when it creates multiple brands. We consider two classes of recommendation. The first one is non-targeted recommendation, where the platform does not use consumer type information. The second one is targeted recommendation, where the platform recommends the list of brands contingent on consumer types. In the privacy environment where the platform does not have consumer preference information, only non-targeted recommendation is possible; in the no privacy environment, both classes of recommendation are possible. We show in Proposition 3 that in privacy equilibrium, although it can filter the brands available to consumers, it would not do so in equilibrium, in order to maintain the informational value of virtual brands. The two-brand equilibrium with full recommendation is the equilibrium under privacy. By contrast, in the no privacy environment, the platform will choose the targeted recommendation by showing only one brand to each consumer according to their types. The creation of multiple brands with different leading products by the two-product firm makes the targeting possible. In such an equilibrium, however, the firm loses its ability to transmit product-variety information to consumers.

3.5.1 Non-targeted recommendation

When the platform does not know consumer information, its strategies are restricted to recommending brands to consumers not dependent on their types. Such non-targeted recommendation by the platform can potentially add value by filtering the information available to consumers and affecting the firm's pricing strategies and consumer traffic through their expectations. If the platform shows one of the brands with equal probability when the two-product firm creates two distinct brands, as argued in Lemma 1, the two-product firm would offer both products for each brand. The randomization of the platform leads to the two-product brand pooling with a single-product brand from consumers' perspective, and the equilibrium outcome will be the same as the single-brand equilibrium in Section 3.3. For the platform, the industry profit is the weighted sum of the single-product firm profit and the two-product firm profit. We have shown in Proposition 1 that both firm types enjoy higher expected profit under the two-brand equilibrium than under the single-brand equilibrium. Therefore, this kind of selective recommendation by the platform is worse than full recommendation.

Alternatively, the platform can show one of the two brands with unequal probability. Such randomization by the platform leads to an equilibrium outcome that is the same as a general single-brand equilibrium in Section 3.3. For instance, if the platform shows the leading-1 brand more often when both the leading-1 and leading-2 brands are available, the two-product firm would find just creating a leading-2 brand and pooling with the j_2 firm is better. The resulting equilibrium outcome will be the same as one of the single-brand equilibria. Because the industry profit is a weighted sum of all three firm types and we already show all three firm types benefit more from the two-brand equilibrium, this class of selective recommendation strategies is also worse for the platform than full recommendation.

A firm's creation of multiple brands, coupled with the platform's full recommendation, effectively communicates information about the product variety to consumers. This reduction in information asymmetry assists in directing consumer traffic, because it enables consumers with heterogeneous preferences to make more informed decisions about whether to explore a particular brand. Consumers would intensify their search efforts when the likelihood of a purchase is higher, and reduce their search intensity when the opposite is true. Should the platform choose not to recommend all available brands, this value derived from information transmission is compromised. Furthermore, as consumers do not observe product prices prior to their search, platform strategies that selectively recommend products cannot directly stimulate consumer search by pushing the firm to lower prices. Hence, the platform benefits from full recommendation. It is important to note the information channel regarding product variety, as outlined here, remains relevant and effective even in the context of a competitive environment involving multiple firms, as we discuss in Section 5.1.

3.5.2 Targeted recommendation

When the platform knows consumer-preference information, it can link the recommendation of brands to consumer types. If the two-product firm just creates one brand, the resulting equilibrium outcome will be the same as one of the single-brand equilibria, which we have shown is worse for the platform. If the two-product firm creates one leading-1 brand and one leading-2 brand, the targeting occurs for the two-product firm when the platform shows each consumer one of the brands. Given that the two-product firm provides both products in each brand as in Lemma 1, the targeted recommendation would base on two cases: (i) The platform shows each consumer the two-product brand leading with her familiar product; and (ii) the platform shows each consumer the two-product brand leading with her unfamiliar product. The platform may also mix these two pure targeted recommendation with non-targeted recommendation. In the main text, we just discuss pure targeted recommendation and assume that the two-product firm still sets prices at (v, v) to better illustrate how targeted recommendation can be beneficial. In the Appendix, we show how the platform can fine tune its mixed targeted recommendation strategies to make the two-product firm still sets prices at (v, v) and how the platform may optimally choose targeted recommendation strategies that involve asymmetric pricing by the two-product firm.

Familiar product: If the platform shows each consumer, the two-product brand leading with her familiar product, consumers cannot tell ex ante whether such a brand also offers their unfamiliar product, and they hold belief $\beta = \gamma/(2-\gamma)$ that it just offers the leading product. For the brand leading with their unfamiliar product, consumers know for sure it just offers this single product. Denote the equilibrium price set by the single-product firm as p^F, we have the equilibrium price p^F given by

$$p^F = \begin{cases} \frac{1}{2} + \frac{\gamma(v-p^F)+(1-\gamma)(1-v)^2}{(2-\gamma)(1-p^F)^2} \in (p^*, v), & \text{if } \gamma > 2 - \frac{1}{\frac{3}{2}-v}, \\ v & \text{if } \gamma \le 2 - \frac{1}{\frac{3}{2}-v}. \end{cases} \tag{4}$$

Compared with the full recommendation equilibrium, as a result of the platform pooling the two-product brand with the single-product brand that consumers are familiar with, the two-product firm suffers from less consumer traffic. In the meantime, as more consumers are attracted by the possibility of wider product varieties, the single-product firm has the leeway to charge higher prices, which in turn leads to less consumer traffic through higher expected prices. As a result, both the single-product firm's profits and the two-product firm's profits are also lower. Therefore, the platform would earn a lower expected revenue by adopting this strategy than full recommendation.

Unfamiliar product: If the platform shows each consumer the two-product brand leading with her unfamiliar product, consumers cannot tell ex ante whether such a brand also offers their familiar product, and hold belief $\beta = \gamma/(2-\gamma)$ that it just offers the leading product. If the j_1 firm sets price at p^U, we have the optimal price p^U given by

$$p^U = \frac{1}{2} + \frac{(2-\gamma)(v-p^U)}{\gamma(1-p^U)^2 + (2-2\gamma)(1-v)^2} \in (p^*, p^P). \tag{5}$$

Compared with the full recommendation equilibrium, as a result of the platform pooling the two-product brand with the single-product brand that consumers are unfamiliar with, the two-product firm gains more consumer traffic. Similar to the familiar product case, the single-product firm would charge higher prices, which leads to less consumer traffic through higher expected prices and lower profits. However, the two-product firm benefits from more consumer traffic and earns a higher profit by pooling with the single-product brand offering consumers' unfamiliar product. It can be shown that the industry profit is higher than the profit in full recommendation equilibrium, so the platform benefits from targeted recommendation.[16]

Combining analyses of the two targeted recommendation strategies, the targeted recommendation does better for the platform than the full recommendation when multiple brands are created. Therefore, we have the equilibrium strategy of the platform summarized in Proposition 3.

Proposition 3. *In the privacy equilibrium, the platform chooses the full recommendation and the two-product firm creates two distinct brands with different leading products. In the no privacy equilibrium, the two-product firm still creates two distinct brands, and the platform chooses the targeted recommendation by recommending each consumer segment a distinct brand.*

When the platform does not have consumer information to targetedly recommend a list of brands (privacy environment), it would maintain an effective communication channel by recommending all brands to all consumers, particularly through the use of two distinct brands that lead with different products from the two-product firm. This approach to reduce information asymmetry results in a Pareto improvement, benefiting both the firm and the consumers, and by extension, the platform itself. When the platform has access to consumer information (no privacy environment), however, the presence of multiple distinct brands also enables targeted recommendation.

[16] When the platform always targetedly recommends the two-product brand leading with a consumer's unfamiliar product, the two-product firm may deviate from the symmetric pricing of (v, v). In the proof of Proposition 3, we show how the platform can optimally mix the targeted recommendation with full recommendation to keep the two-product firm charge prices (v, v) and earn a higher profit than full recommendation equilibrium. We also give an example how the targeted recommendation of the two-product brand leading with a consumer's familiar product can be optimal in some cases.

Targeted recommendation, by pooling the two-product brand with a specific type of single-product brand, leads to more "effective" consumer engagement. Although this approach risks losing potential visits to the single-product firm, it attracts more consumers to search two-product brands, whereby consumers are more inclined to make a purchase. Through targeted recommendation, the platform effectively guides consumers to intensify their search when the firm is better at making profits, that is, when the firm offers both products. It is worthwhile to mention that the creation of multiple brands with distinct leading products by the two-product firm makes targeting possible for the platform. Compared with the single-brand equilibrium, the use of two distinct brands by the two-product firm, together with targeted recommendation by the platform, makes the beliefs of heterogeneous consumers diverge prior to their search and hence affect their subsequent search decisions. In the extension, we explore how targeted recommendations by the platform can be advantageous through an alternative channel in a competitive environment with multiple firms.

4 Banning Identical-Menu Virtual Brands

A policy-relevant question we would like to answer is whether the platform can achieve better outcomes through some restrictions. More specifically, how would the platform banning the identical-menu virtual brands affect the firm, consumers and the platform? Specifically, we consider that brands cannot offer the same product in different brands if identical-menu virtual brands are banned. With the ban, the two-product firm is restricted to either create two brands each offering a distinct product, or one single brand with both products. For one single brand case, the equilibrium will be the single-brand equilibrium we analyzed above, which is no better than the two-brand equilibrium outcome or the targeted recommendation outcome for the platform.

For the two-distinct-brand case, we first consider the full recommendation by the platform, and later show doing so is indeed optimal for the platform. With each brand offering only a single product, the leading product tells the exact product variety. In the meantime, the number of brands reveals the firm type to consumers. The single-product firm would operate as in the two-brand equilibrium by setting price to be p^*. For the two-product firm, as the two brands offer distinct products, the search order of each consumer does matter in equilibrium. For instance, if each consumers visits the brand offering her familiar product first, the firm would set the price to be v, extracting all the surplus. Expecting this outcome, she would not search first her familiar-product brand in equilibrium. If each consumer visits the her unfamiliar-product brand first, the firm would set the price to be $1/2$ to maximize profit. The consumer surplus conditional on visit is $1/8$. Consumers indeed search the unfamiliar-product brand first if $v < 5/8$. Otherwise, consumers would instead search the familiar-product brand first, and the equilibrium breaks down.

The above argument, however, is not complete, because some consumers with low search cost may search the second brand given that these two brands now offer different products. We consider symmetric equilibrium here. Let the two-product firm sets prices to be $\hat{p}$ for both products in equilibrium. Consumers would rationally expect this decision, and the expected surplus of visiting the unfamiliar-product brand is $(1 - \hat{p})^2/2$, while the expected surplus of visiting the familiar-product brand is $v - \hat{p}$. Consumers would indeed search the unfamiliar-product brand first if

$$\frac{(1-\hat{p})^2}{2} \geq v - \hat{p} \iff \hat{p} > \sqrt{2v-1}.$$

Consumers with high search cost would only search the unfamiliar-product brand and decide to buy or not, and the highest search cost to ever start searching is $c = (1-\hat{p})^2/2$. Consumers with lower search cost will continue searching the familiar-product brand if the realized utility of the unfamiliar product u is not high enough. As consumers know the exact utility before they search the familiar-product brand and rationally expect the equilibrium price, the decision to continue searching the familiar-product brand is the same as buying from it in equilibrium. We assume that if consumers see an off-path price p for the first brand they visit, they will adjust their off-path expectation of the product price of the second brand to be the same price p.[17]

When two distinct brands are available, consumers with search cost below $(1-\hat{p})^2/2$ will initiate search and start with their unfamiliar-product brand. For the j_{12} firm, the revenue comes from two sources: those who find them particularly fond of the unfamiliar product (high u) and those who are not so fond but have low search cost to continue search and buy from the familiar-product brand. The expected profit for the j_{12} firm to charge price p is

$$\begin{aligned}
&\Pr\left[c \leq \frac{(1-\hat{p})^2}{2}\right]\left(\Pr\left[c > v-p \;\&\; u > p \middle| c \leq \frac{(1-\hat{p})^2}{2}\right] + \Pr\left[c \leq v-p \middle| c \leq \frac{(1-\hat{p})^2}{2}\right]\right)p \\
=&\frac{p}{\bar{c}}\left(\frac{(1-\hat{p})^2}{2} - \left(\frac{(1-\hat{p})^2}{2} - (v-p)\right)p\right).
\end{aligned}$$

Therefore the optimal price is given by

$$\hat{p} = \frac{1}{2} + \frac{\hat{p}(2v-3\hat{p})}{(1-\hat{p})^2}. \tag{6}$$

To sustain the equilibrium search sequence whereby consumers search their unfamiliar-product brand first, we need $\hat{p} > \sqrt{2v-1}$, which implies $v < 2-\sqrt{2}$ and $\hat{p} \in (2v/3, \sqrt{2}-1)$. We show in the proof of Proposition 4 that the two-product firm earns a higher profit than under the two-brand equilibrium when using virtual brands is not restricted. By induction, for the two-product firm, creating two brands, each offering a distinct product, also yields higher profit than creating just a single brand and hence would be the equilibrium strategy. For the platform, the industry profit is

$$\gamma\frac{{p^*}^2(1-p^*)^2}{4\bar{c}} + (1-\gamma)\frac{\hat{p}((1-\hat{p})^2+2\hat{p}^2)}{4\bar{c}}.$$

Because the profit of the single-product firm stays the same, we can easily show the platform also earns a higher profit than the two-brand equilibrium and hence would find it optimal to choose full

[17] Alternatively it can be interpreted as we are focusing on the symmetric equilibrium that the firm always chooses the same price for both products. The reasoning is that each brand would attract one of the two consumer segments with equal population to search first. If the two-product firm finds it profitable to choose a off-path price p for one of the brands, it should also be profitable for the other brand.

recommendation.

Proposition 4. *With ban on identical-menu virtual brands, when $v \leq 2-\sqrt{2}$, the platform chooses full recommendation, while the two-product firm would create two distinct brands, each offering a different product, benefiting itself, the platform, and consumers, in comparison with the equilibrium outcome without the ban.*

When a two-product firm introduces two separate brands, each dedicated to a single product, it remarkably limits its own capacity to leverage consumers' preference flexibility for profit extraction. This strategy compels the firm to adopt more competitive pricing, because consumers naturally gravitate toward the brand offering them higher expected surplus and providing the product they're less familiar with. This anticipation of lower prices intriguingly draws a larger customer base, creating a mutual benefit scenario for both consumers and the two-product firm. However, realizing this positive outcome hinges on the intervention by the platform: the prohibition of virtual brands with identical menus. Without this ban, a two-product firm would be inclined to offer both products under each brand, aiming to inflate prices and maximize profit extraction from consumers by taking advantage of consumers' flexible preferences.

When each brand offers a different product, the search sequence matters and consumers would search the brand offering their unfamiliar product first. Although diversification seems to make the search less efficient by increasing the overall search cost for consumers, surprisingly, both the two-product firm's profit and consumers' surplus are higher than those under no-ban equilibrium. With consumers self-selecting to search the unfamiliar-product brand first, the firm needs to set a lower price. First, consumers have a higher expected surplus from searching their unfamiliar product, and the optimal price facing this kind of demand is lower. Second, additional information about price is revealed after consumers visit one of the two brands created by the two-product firm. The two-product firm can use a lower price to credibly persuade consumers to continue searching the second brand in case the first one does not provide a desirable option. However, this equilibrium is only possible when the utility of the familiar product is not so high that consumers are willing to search the brand offering their unfamiliar product first. Without the ban, a "hold-up" problem arises because consumers do not observe actual product prices prior to their search, while the firm has incentives to raise prices conditional on being searched, which makes the search intensity inefficiently low. The ban pushes different brands to offer distinct products and makes the order in which consumers search among these brands pivotal. This change motivates the multi-product firm to lower prices even when consumers cannot observe them prior to their search, which, in turn, increases consumer traffic and mitigates the hold-up problem. By making it hard for the firm to exploit consumers, the ban aligns the interests of consumers and the firm, fostering a win-win situation.

To consider the full equilibrium, we need to solve the equilibrium when $v > 2 - \sqrt{2}$ and check the platform's incentives. So far, we have analyzed the equilibrium where the platform chooses full recommendation. Alternatively, the platform may show only one of the brands when the two-product firm creates two brands. Three general cases exist: (i) The platform targetedly shows consumers their familiar-product brand; (ii) the platform targetedly shows consumers their unfamiliar-product brand; and (iii) the platform, without targeting, randomly shows one of the brands to each consumer with equal probability.[18] We show an equilibrium cannot exist whereby the two-product firm creates two distinct brands with different products while the platform selectively recommends one brand. When $v > 2 - \sqrt{2}$, the only equilibrium exists where the two-product firm creates a single brand with both products and chooses the leading product with equal probability and the platform still shows all available brands to consumers.

Proposition 5. *When identical-menu virtual brands are banned, the platform would choose full recommendation. If $v \leq 2-\sqrt{2}$, the two-product firm would create two distinct brands with different products; if $v > 2-\sqrt{2}$, the two-product firm would just create one brand with both products, equally likely to be leading-1 or leading-2.*

The ban of identical-menu virtual brands together with full recommendation by the platform maintains the same informational value of virtual brands by showing product variety to consumers prior to search. In fact, the benefits of recommending all available brands go beyond such informational value because now the two brands offer distinct products and consumers sometimes get material benefit when searching the second brand. At the same time, the two-product firm would lower prices to persuade consumers to continue search. Therefore, not selectively recommending brands is in the platform's best interest. As we said, such a win-win situation only occurs if v is relatively small. Otherwise, consumers would search the brand offering their familiar product first and the firm would want to raise prices, making the equilibrium collapse. When $v > 2 - \sqrt{2}$, the two-product firm would create only one brand with both products and the platform would recommend this single brand. Potentially, the two-product firm can still create two brands with distinct products if the platform targetedly recommends an unfamiliar-product brand to each consumer. This targeted recommendation can solve the hold-up problem and attracts more consumers to visit, but then the two-product firm would be less able to turn visits into purchase with just one product for each consumer. Therefore, when v is relatively large, the ban would turn the equilibrium into the single-brand equilibrium, which is worse for both consumers and the firm.

5 Extensions

We consider several extensions for our main model here and show how the informational channel of product variety through virtual brands plays out in other settings and how some new insights emerge. Proofs of results presented in this section are provided in the Online Appendix.

[18] We already show in the proof of Lemma 2 that the equal-probability randomization is the only possible non-targeted pooling equilibrium.

5.1 Competing firms

We can extend our results to competition among firms. To get rid of the returning demand, assume there are infinitely many firms.[19] Following the sequential search literature, assume search cost c is constant across all consumers and low enough for consumers to initiate search. Here the assumption that consumers know the ownership of each brand is not necessary. For each leading-brand type, infinitely many brands are created by different firms. Information can be communicated through the platform's selective recommendation to consumers instead of being directly communicated through observing multiple brands from the same firm. For example, if two-product firms create two brands with distinct leading products, the platform can observe these different brands and select a list of two-product brands offered by two-product firms. In addition, it can commit to displaying for each consumer a list of two-product brands with the leading product being her familiar product only. Thus, consumers will still know they are searching a list of two-product brands even if they just see one brand from each firm.

To simplify our analysis, we restrict the platform to present either one brand, representing for monopoly, or a list of infinite brands, representing for competition. For the choice of brands, we restrict the platform to choosing just one type of brand for each consumer. If one brand type offers the highest expected profit, the platform would just choose that type for all brands it presents. We focus on the symmetric equilibrium where the same type of firms use the same strategy.

If the platform recommends to consumers just one brand or a list of brands offering only their familiar product, the firms would all charge prices v and no consumer would search for any $c > 0$ as in Diamond (1971). If each consumer is presented a list of brands offering only an unfamiliar product, then for a consumer with search cost c, the optimal strategy is to buy immediately when the value of the product $u > 1 - \sqrt{2c}$, and continue search otherwise. The firms would charge price $\sqrt{2c}$ for their products, with expected profit $2c$. For the platform, the industry profit is $\sqrt{2c}$. If each consumer is presented a list of brands offering both products, with the same leading product, firms can charge different prices for the two products. Without loss of generality, we consider type-L consumers, and leading-1 brand. In the symmetric equilibrium, all these firms charge the same price $p_1^\infty = \frac{5v-1}{4} + \frac{2c}{4(1-v)}$ for product 1 and the same price $p_2^\infty = \frac{3+v}{4} - \frac{2c}{4(1-v)}$ for product 2 in their leading-1 brands. Type-L consumers would buy product 1 if $u < 1 - \frac{4c}{1-v}$ and product 2 otherwise from the first brand she randomly chooses from the list. This equilibrium is only feasible when $c < (1-v)^2/8$. Otherwise, the equilibrium will be that $p_1^\infty = v$, $p_2^\infty = (1+v)/2$, and consumers would not search any brand, expecting non-positive surplus with positive search cost.[20]

To find the full equilibrium, we focus on four types of strategies for the platform: (1) recommending just one brand offering unfamiliar product only, (2) recommending just one brand offering both products from the same firm, (3) recommending a list of brands offering unfamiliar products only, and (4) recommending a list of brands offering both products.[21] We have the following

[19] Consumers can never exhaust the list of brands, so when they decide to continue search, they find it optimal not to come back and buy from a previously searched brand. See also Jiang and Zou (2020) and Zhong (2023).

[20] The analyses are presented in the proof of Proposition 6.

[21] Other possible equilibria may generate the same payoffs as these equilibria under some conditions. In case (4),

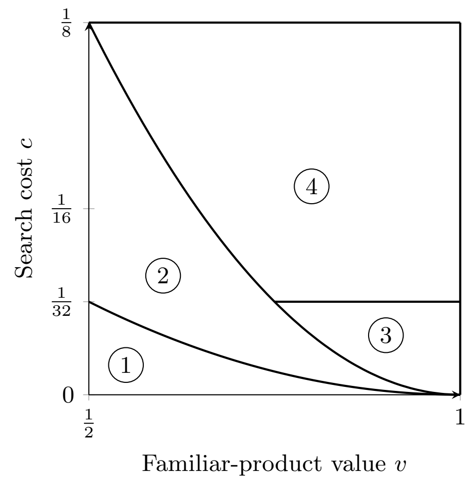

Figure 4: Equilibria with competing firms

Note: The numbered labels in the graph refer to the cases of equilibria in Proposition 6.

proposition.

Proposition 6. *The equilibria for different combinations of v and c are as follows.*

1. *When $c < (1-v)^2/8$, the platform would present a single brand offering both products with different leading products targeting different consumer segments (targeted recommendation).*
2. *When $(1-v)^2/8 \leq c \leq (1-v)^2/2$, the platform would present a single brands offering both products with a random leading product for all consumers (non-targeted recommendation).*
3. *When $(1-v)^2/2 < c \leq 1/32$, the platform would present a single brand offering only consumers' unfamiliar product (targeted recommendation).*
4. *When $\max\{(1-v)^2/2, 1/32\} < c < 1/8$, the platform would present a list of brands offering only consumers' unfamiliar product (targeted recommendation).*

In comparison with our main model featuring a single firm, the variety information still plays a pivotal role. The result that two-product firms would create two brands with distinct leading products still holds in most cases with competition. The difference is that the information regarding product variety conveyed through multiple brands can be mediated by the selective recommendation of the platform. When the platform commits to recommending two-product brands, consumers observing just one brand created by a two-product firm can still infer the product variety. When the search cost is low enough, the creation of such virtual brands together with the selective recommendation increases firm and industry profit through better product matching and more sophisticated pricing strategies. As a result, whenever possible, the platform would recommend two-products brands created by two-product firms. In addition, the platform's targeted recommendation

we focus on targeted recommendation so that each consumer segment receives a (list of) specific brand(s). We use these equilibria to illustrate our key points.

based on consumer preference type can facilitate better profit extraction from consumers. Thus, when the search cost is low and consumers would initiate search anyway, targeted recommendation brings more profit for the platform.

The extension also differs with our main model in that we assume the search cost is fixed. Competition inevitably lowers the industry profit, so with a fixed search cost, the platform would recommend to each consumer brands from a single firm only instead of from a list of firms to maximize profit if possible. However, we would expect that when search cost is random, competition may enhance industry profits and be beneficial to the platform by attracting consumers with a higher search cost to search.

With continuous firms, the ban on identical-menu brands essentially breaks the two-product brands created by two-product firms into single-product brands. In absence of uncertainty about the product variety in any brand, the ban would play a similar role as providing consumers a list of single-product brands. When the search cost is constant, the ban cannot encourage more consumer visits and hence would not change platform strategies and the equilibrium. If the search cost is random as in our main model, we can expect the ban to encourage more consumer visits and push down product prices, which could be beneficial to the platform, when it is optimal for the platform to recommend brands from a two-product firm.

5.2 Inferior familiar product

In the main model, we restrict the familiar product to be superior to the unfamiliar product in terms of expected consumer valuation, that is, $v > 1/2$. To complement our main analysis, this section studies the case in which $v < 1/2$; that is, the familiar product offers consumers a lower expected utility than their unfamiliar product. We start with analyzing the firm strategies. An immediate result is that the optimal product prices chosen by the single-product firm would either be v or $1/2$. The j_1 firm, given a fraction α of consumers being type-L, would choose p_1 to maximize

$$\begin{cases} \alpha p_1 + (1-\alpha)(1-p_1)p_1 & \text{if } p_1 \leq v, \\ (1-\alpha)(1-p_1)p_1 & \text{if } p_1 > v. \end{cases}$$

We have the optimal price given by

$$\begin{cases} v & \text{if } \alpha \geq 1 - \frac{4v}{1+4v^2}, \\ \frac{1}{2} & \text{if } \alpha < 1 - \frac{4v}{1+4v^2}. \end{cases}$$

The fraction α is endogenously determined by the search decisions of consumers. Because the familiar product has a lower value v, conditional on consumer visit, the single-product firm would either serve both types of consumers by setting price to be v or charge the monopoly price $1/2$ serving only consumers unfamiliar with it. As consumers cannot earn non-positive surplus from buying their familiar product in any case, they would not visit a brand if for sure it provides just their familiar product. The expected surplus of a consumer visiting a brand with only her unfamiliar product is just $1/8$. Next, by solving the equilibrium and comparing the firm profit under different cases, we have the two-product firm's equilibrium pricing strategy in the separating equilibrium under full recommendation given in the following lemma.

Lemma 3. *In the two-brand equilibrium, the two-product firm would create two brands with distinct leading products. The optimal pricing strategy is to charge both brands* $(1/2, 1/2)$ *if* $v \leq 1 - \sqrt{6}/3$, *charge both* $(v, (1+v)/2)$ *and* $((1+v)/2, v)$ *with equal probability if* $1 - \sqrt{6}/3 < v \leq 1/3$, *and charge both* (v, v) *if* $1/3 < v < 1/2$.

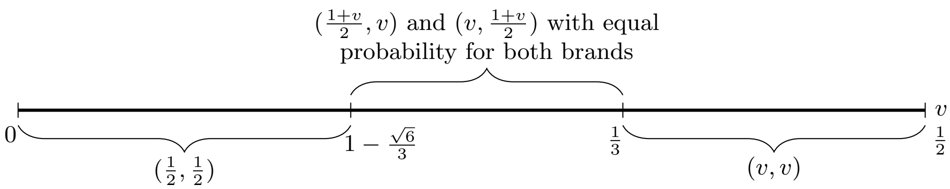

Figure 5: Pricing strategies of the two-product firm with inferior familiar product

In the two-brand equilibrium, the single-product firm would charge price $1/2$ for its sole product and only consumers unfamiliar with this product would visit its brand. By contrast, consumers always have a higher expected surplus from visiting a two-product brand. Therefore, the two-product firm has no incentive to mimic the single-product firm by creating just one brand. We can extend our previous results here that the two-product firm would create two brands with distinct leading products to show its product variety to consumers. The creation of multiple brands increases both its own profit and consumer surplus. For the full equilibrium, we have the following proposition.

Proposition 7. *In equilibrium under* $v < 1/2$, *the platform would choose targeted recommendation if* $v \in (0, 1 - \sqrt{6}/3)$ *and* $\gamma \in ((2 - v^2)/(3 - v^2), 1)$, *and choose full recommendation otherwise. The two-product firm would create two brands with different leading products. By creating virtual brands, the two-product firm increases its own profit while benefiting both types of consumers.*

When the familiar product is inferior to the unfamiliar product, we extend our main finding that the two-product firm would create two brands with different leading products to communicate information about its product variety. This reduction in information asymmetry also benefits consumers so that they search more when they are more likely to buy. In most cases, the platform would also facilitate such communication to guide consumers to search more when they are more likely to purchase.

The ban, however, would not benefit the two-product firm nor consumers with inferior familiar product. In the main model with $v > 1/2$, consumers will first search the brand with their unfamiliar product and their self-selection "forces" the firm to charge a lower price. In addition, the firm can use a lower price to persuade consumers to continue searching the second brand and buy the familiar product if the unfamiliar product is not a good match. With $v < 1/2$, however, the unfamiliar product offers a higher expected value and a higher monopoly price than serving just the familiar product. When the identical-menu virtual brands are banned, the two-product firm would just

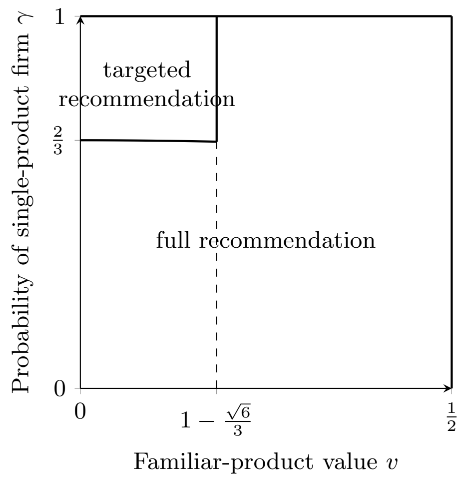

Figure 6: Equilibria under inferior familiar product ($v < 1/2$)

cater the needs of unfamiliar consumers and set price to 1/2 for each product. Consumers would just search the brand with their unfamiliar product. The ban would not lower the product prices and hence cannot induce more consumer traffic. As a result, the platform cannot use the ban to achieve better equilibrium profits.

5.3 Alternative incentive scheme of the platform

In our model, the platform charges a percentage fee on the total revenue, so the platform shares with the firm the incentive to maximize firm profit, with the mild difference being that the platform tries to maximize the industry profit across all firm types. Another common scenario is that the platform charges a fixed fee per transaction, so its objective is to maximize the total trade volume. We show our results extend to this alternative incentive scheme of the platform.

Proposition 8. *When the platform charges a fixed fee per transaction instead of a percentage fee, the equilibria stay the same. The two-product firm creates two brands with the same menu but distinct leading products; the platform chooses full recommendation in the privacy equilibrium, and targeted recommendation in the no-privacy equilibrium. With ban of identical-menu virtual brands, the platform chooses full recommendation, and the two-product firm would create two distinct brands with different products when $v \leq 2-\sqrt{2}$, and would create one brand with both products, equally likely to be leading-1 or leading-2 when $v > 2-\sqrt{2}$.*

Consumers choose whether to visit a brand based on their expected surplus, and hence, the consumer surplus directly determines the transaction volume and affects firm revenue. Therefore, the platform needs to consider both consumer surplus and firm profits under both the total-revenue optimization and transaction-volume optimization. Since the reduction in information asymmetry through virtual brands benefits both consumers and the firm, the platform would still facilitate such information communication by presenting all brands to all consumers when targeting is not possible. Similar to before, targeting is better at extracting surplus when the two-product firm creates two brands with distinct leading products and the platform uses them to target consumers.

6 Conclusion

On online marketplaces and digital platforms, the brand management strategies keep evolving. Companies are increasingly diversifying their online footprint by establishing multiple virtual brands, often offering identical menu of products. Our research studies how creating multiple virtual brands may open up a novel informational channel informing consumers about the firm's product variety, which differs from the traditional umbrella-branding mechanism of product quality signaling.

We develop a model in which a firm with uncertain product variety manages multiple brands. A pivotal discovery of our research is that virtual brands can act as a crucial communication tool for multi-product firms, effectively broadcasting the variety of their product range. The resulting decrease in information asymmetry between the firm and consumers empowers consumers to refine their search strategies, intensifying their search efforts when the expected surplus and purchase likelihood is high. Surprisingly, we found that this dissemination of information benefits not only consumers but also the single-product firm that does not employ such strategies.

In addition, our exploration of the policy prohibiting identical-menu virtual brands suggests that it could lead to a Pareto improvement for both the firm and consumers by inducing more consumers to search brands on the platform. The ban may push different brands to offer distinct products and makes the order in which consumers search among these brands pivotal. This change motivates the multi-product firm to lower prices even when consumers cannot observe them prior to their search, which, in turn, increases consumer traffic. However, this policy warrants careful consideration, as it may lower overall welfare if it leads firms to consolidate their products under a single brand, thereby closing off this informational channel through virtual brands.

We also highlight the role of the platform in setting its recommendation strategies and how these equilibrium strategies differ between a privacy environment, where the platform does not know the consumer's preference type, and a no-privacy environment, where the platform does have this knowledge. In the privacy equilibrium, the platform would sustain the informational channel of communicating product variety using virtual brands by recommending all brands to all consumers, because it results in a win-win-win equilibrium for the two-product firm, consumers and the single-product firm. In the no-privacy equilibrium, the creation of multiple brands with distinct leading products by the firm also makes targeting possible, and the platform would indeed choose targeted recommendation. The targeted recommendation pools each brand created by the two-product firm with a single-product brand featuring the same leading product. The targeted recommendation takes away some consumer traffic from the single-product firm, but brings more consumer visits to the two-product firm that is better at selling to consumers. The industry profit increases as a result.

Several promising directions for future research emerge when considering brands as a tool for communicating product variety information. First, we show our main insights can be extended to competition with infinite firms in the extension. In an oligopoly market with competing firms and uncertain brand ownership, however, the process of consumer belief updating may become more evolved. The prior knowledge of brand ownership and potential consumer behavioral bias can alter communication channels, impacting consumer search decisions and firm competition. Second, our model's consumer heterogeneity is currently based on which product consumers are familiar with. An alternative scenario could involve both products having uncertain values prior to search, with heterogeneity arising from different consumers perceiving different products to have a higher expected utility. In this setting, targeted recommendations from the platform might enhance welfare due to the stronger force of product matching.

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References

[1] Wilfred Amaldoss and Sanjay Jain. "Branding conspicuous goods: An analysis of the effects of social influence and competition". *Management Science* 61.9 (2015), pp. 2064–2079.

[2] Wilfred Amaldoss and Woochoel Shin. "Multitier store brands and channel profits". *Journal of Marketing Research* 52.6 (2015), pp. 754–767.

[3] Maria Arbatskaya. "Ordered search". *The RAND Journal of Economics* 38.1 (2007), pp. 119–126.

[4] Mark Armstrong and John Vickers. "Multibrand price dispersion". *Working Paper* (2024).

[5] Mark Armstrong, John Vickers, and Jidong Zhou. "Prominence and consumer search". *The RAND Journal of Economics* 40.2 (2009), pp. 209–233.

[6] Bart J Bronnenberg, Jean-Pierre Dubé, and Sridhar Moorthy. "The economics of brands and branding". In: *Handbook of the Economics of Marketing*. Vol. 1. Elsevier, 2019, pp. 291–358.

[7] Luis MB Cabral. "Stretching firm and brand reputation". *RAND Journal of Economics* (2000), pp. 658–673.

[8] Peter A Diamond. "A model of price adjustment". *Journal of Economic Theory* 3.2 (1971), pp. 156–168.

[9] Anthony Dukes and Lin Liu. "Online shopping intermediaries: The strategic design of search environments". *Management Science* 62.4 (2016), pp. 1064–1077.

[10] Michael D Grubb and Ryan Westphal. "The illusion of competition". *Working Paper* (2024).

[11] Liang Guo and Yue Wu. "Consumer deliberation and quality signaling". *Quantitative Marketing and Economics* 14 (2016), pp. 233–269.

[12] Andrei Hagiu and Bruno Jullien. "Why do intermediaries divert search?" *The RAND Journal of Economics* 42.2 (2011), pp. 337–362.

[13] Baojun Jiang and Tianxin Zou. "Consumer search and filtering on online retail platforms". *Journal of Marketing Research* 57.5 (2020), pp. 900–916.

[14] T Tony Ke, Song Lin, and Michelle Y Lu. "Information design of online platforms". *HKUST Business School Research Paper* 2022-070 (2022).

[15] T Tony Ke, Jiwoong Shin, and Jungju Yu. "A model of product portfolio design: Guiding consumer search through brand positioning". *Marketing Science* 42.6 (2023), pp. 1101–1124.

[16] Richard E Kihlstrom and Michael H Riordan. "Advertising as a signal". *Journal of Political Economy* 92.3 (1984), pp. 427–450.

[17] Benjamin Klein and Keith B Leffler. "The role of market forces in assuring contractual performance". *Journal of Political Economy* 89.4 (1981), pp. 615–641.

[18] Dmitri Kuksov. "Brand value in social interaction". *Management Science* 53.10 (2007), pp. 1634–1644.

[19] Paul Milgrom and John Roberts. "Price and advertising signals of product quality". *Journal of Political Economy* 94.4 (1986), pp. 796–821.

[20] Sridhar Moorthy. "Can brand extension signal product quality?" *Marketing Science* 31.5 (2012), pp. 756–770.

[21] Phillip Nelson. "Information and consumer behavior". *Journal of Political Economy* 78.2 (1970), pp. 311–329.

[22] Serdar Sayman, Stephen J Hoch, and Jagmohan S Raju. "Positioning of store brands". *Marketing Science* 21.4 (2002), pp. 378–397.

[23] Fiona Scott Morton and Florian Zettelmeyer. "The strategic positioning of store brands in retailer–manufacturer negotiations". *Review of Industrial Organization* 24 (2004), pp. 161–194.

[24] Carl Shapiro. "Premiums for high quality products as returns to reputations". *The Quarterly Journal of Economics* 98.4 (1983), pp. 659–679.

[25] David A Soberman and Philip M Parker. "The economics of quality-equivalent store brands". *International Journal of Research in Marketing* 23.2 (2006), pp. 125–139.

[26] Steven Tadelis. "What's in a Name? Reputation as a Tradeable Asset". *American Economic Review* 89.3 (1999), pp. 548–563.

[27] Hal R Varian. "A model of sales". *The American Economic Review* 70.4 (1980), pp. 651–659.

[28] Birger Wernerfelt. "Brand loyalty and market equilibrium". *Marketing Science* 10.3 (1991), pp. 229–245.

[29] Birger Wernerfelt. "Umbrella branding as a signal of new product quality: An example of signalling by posting a bond". *The RAND Journal of Economics* (1988), pp. 458–466.

[30] Asher Wolinsky. "True monopolistic competition as a result of imperfect information". *The Quarterly Journal of Economics* 101.3 (1986), pp. 493–511.

[31] Jungju Yu. "A model of brand architecture choice: A house of brands vs. a branded house". *Marketing Science* 40.1 (2021), pp. 147–167.

[32] Zemin Zhong. "Platform search design: The roles of precision and price". *Marketing Science* 42.2 (2023), pp. 293–313.

[33] Bo Zhou and Tianxin Zou. "Competing for recommendations: The strategic impact of personalized product recommendations in online marketplaces". *Marketing Science* 42.2 (2023), pp. 360–376.

7 Proofs

Proof of Lemma 1. Say, a two-product firm creates a leading-1 brand, and a fraction $\alpha \in [0,1]$ of consumers visiting this brand are type-L. Conditional on visit, if the firm includes only product 1 in the brand, it can choose the product-1 price to maximize the profit as $\alpha p + (1-\alpha)p(1-p)$. The optimal price $p(\alpha)$ is

$$p(\alpha) = \begin{cases} \frac{1}{2(1-\alpha)} & \text{if } \alpha < 1 - \frac{1}{2v}, \\ v & \text{if } \alpha \geq 1 - \frac{1}{2v}. \end{cases}$$

If the firm creates a brand with both products, it can choose product-1 and product-2 prices $p, p' \leq v$ so that both types of consumers would buy one of the products for sure and maximize the profit as

$$\begin{aligned} &\alpha \left(p \Pr[v - p \geq u - p'] + p' \Pr[v - p < u - p']\right) + (1-\alpha) \left(p \Pr[u - p \geq v - p'] + p' \Pr[u - p < v - p']\right) \\ =& -(p - p')^2 + (2\alpha - 1)v(p - p') - \alpha(p - p') + p. \end{aligned}$$

For the equality, it is easy to see that $v - p + p' > 0$, and easy to verify that the profit is increasing in p if $v - p + p' > 1$. Therefore, $v - p + p', v - p' + p \in [0,1]$. We get optimal product prices are $p = p' = v$ regardless of α, and the optimized profit is v. It is easy to check that this profit is higher than the profit from creating a single-product brand. Alternatively, if $p, p' > v$, only consumers unfamiliar with the product would buy and the firm finds it optimal to set prices to be $p = p' = 1/2$, which forms a contradiction. The last possibility is that one of the prices is strictly above v while the other is weakly below v. For instance, if $p \leq v < p'$, the firm would maximize the profit as

$$\begin{aligned} &\alpha \left(p \Pr[v - p \geq u - p'] + p' \Pr[v - p < u - p']\right) + (1-\alpha) p \Pr[u - p \geq 0] \\ =& -\alpha(p' - p)^2 + \alpha(1 - v)(p' - p) - (1-\alpha)p^2 + p. \end{aligned}$$

The optimal prices are

$$\begin{cases} \left(\frac{1}{2(1-\alpha)}, \frac{1}{2(1-\alpha)} + \frac{1-v}{2}\right) & \text{if } \alpha < 1 - \frac{1}{2v}, \\ \left(v, \frac{1+v}{2}\right) & \text{if } \alpha \geq 1 - \frac{1}{2v}, \end{cases}$$

It is easy to check that this profit is also higher than the profit from creating a single-product brand. In addition, comparing the profits from these possible strategies of the two-product firm, we have the optimal pricing strategies given by

$$\begin{cases} (v, v) & \text{if } \frac{(1-v)^2}{4v^2 + (1-v)^2} \leq \alpha \leq \frac{4v^2}{4v^2 + (1-v)^2}, \\ \left(v, \frac{1+v}{2}\right) & \text{if } \alpha > \frac{4v^2}{4v^2 + (1-v)^2} > 0.8, \\ \left(\frac{1+v}{2}, v\right) & \text{if } \alpha < \frac{(1-v)^2}{4v^2 + (1-v)^2} < 0.2. \end{cases}$$

Notice, however, that the asymmetric pricing is very hard to sustain in equilibrium, because the fraction α depends on consumers' expectation of surplus. When the prices are set as $(v, (1+v)/2)$, type-L consumers would expect lower consumer surplus than type-R consumers, so the resulting fraction α is more likely to be very small, making the prices $(v,(1+v)/2)$ non-optimal, similarly for $((1+v)/2,v)$. An immediate result is that in the separating equilibrium, the two-product firm would not use asymmetric pricing. In addition, setting prices $(v,(1+v)/2)$ and $(1+v)/2,v)$ with equal probability yields less profit than (v,v). □

Proof of Lemma 2. If each firm can create only one brand, first consider a semi-pooling case where the two-product firm establishes only a leading-1 brand. Consider first that the two-product firm sets prices (v,v). For a leading-2 brand created by the j_2 firm, consumers are certain that it provides only product 2 before they visit. As consumers rationally anticipate the price, j_2 firm faces the same problem as in two-brand equilibrium and would set $p_2 = p^*$ with the same expected profit. Let the optimal price set by j_1 firm be p^{SP}. For a leading-1 brand, from consumers' perspective, the probability of it being a single-product brand is $\beta^1 = \gamma/(2-\gamma)$, so the expected payoff for type-L consumers visiting this brand is $\frac{\gamma}{2-\gamma}(v-p^{SP}) + \frac{2-2\gamma}{2-\gamma}\frac{(1-v)^2}{2}$, and the expected profit for type-R consumers visiting the brand is $\frac{\gamma}{2-\gamma}\frac{(1-p^{SP})^2}{2} + \frac{2-2\gamma}{2-\gamma}\frac{(1-v)^2}{2}$. As the leading-1 brand may provide just product 1, type-L and type-R consumers have different expected payoffs, and hence visit such a brand with different probabilities. The j_1 firm, facing unequal proportion of two types of consumers, sets the optimal price to satisfy

$$p^{SP} = \begin{cases} \frac{1}{2} + \frac{\gamma(v-p^{SP})+(1-\gamma)(1-v)^2}{\gamma(1-p^{SP})^2+(2-2\gamma)(1-v)^2} \in (p^P, v), & \text{if } \gamma > \frac{4-4v}{3-2v}, \\ v & \text{if } \gamma \leq \frac{4-4v}{3-2v}. \end{cases}$$

When the proportion of single-product firm γ decreases, it sets a higher price based on the same reasoning as in the single-brand (pooling) equilibrium in the main text.

The semi-pooling and pooling equilibria are special cases of more general single-brand equilibria where the two-product firm is restricted to only hold a single brand and chooses product 1 as the leading product with probability $\eta \in [0,1]$. The semi-pooling case is $\eta = 1$ and the pooling one we discussed in the main text is $\eta = 1/2$. Given η, consumers assign probability $\beta^1 = \gamma/(\gamma+2(1-\gamma)\eta)$ to a leading-1 brand selling just product 1 and probability $\beta^2 = \gamma/(\gamma+2(1-\gamma)(1-\eta))$ to a leading-2 brand just selling product 2. j_i firm would just create a brand with its only product and optimally charge price $\tilde{p}_i$ for product i, $i \in \{1,2\}$. Type-L consumers' expected surplus of visiting a leading-i brand is $\beta^i(v-\tilde{p}_i) + (1-\beta^i)\frac{(1-v)^2}{2}$. Type-$R$ consumers' expected surplus of visiting a leading-i brand is $\beta^i\frac{(1-\tilde{p}_i)^2}{2} + (1-\beta^i)\frac{(1-v)^2}{2}$. The j_i firm maximizes profit from two types of consumers by choosing $\tilde{p}_i$ that satisfies

$$\tilde{p}_i = \begin{cases} \frac{1}{2} + \frac{\beta^i(v-\tilde{p}_i)+(1-\beta^i)(1-v)^2/2}{\beta^i(1-\tilde{p}_i)^2+(1-\beta^i)(1-v)^2} & \text{if } \beta^i > 2(1-v), \\ v & \text{if } \beta^i \leq 2(1-v). \end{cases}$$

It is easy to show that $\tilde{p}_i$ is decreasing in β^i. For the two-product firm, the expected profit of mixing with probability η is

$$\frac{\eta}{2\bar{c}}\left(\beta_1(v-\tilde{p}_1)+\beta_1\frac{(1-\tilde{p}_1)^2}{2}+(1-\beta_1)(1-v)^2\right)v+\frac{1-\eta}{2\bar{c}}\left(\beta_2(v-\tilde{p}_2)+\beta_2\frac{(1-\tilde{p}_2)^2}{2}+(1-\beta_2)(1-v)^2\right)v.$$

It can be shown that $(\beta^i(v-\tilde{p}_i)+\beta^i\frac{(1-\tilde{p}_i)^2}{2}+(1-\beta^i)(1-v)^2$ is strictly increasing in β^i and β^1 (β^2) is strictly decreasing (increasing) in η, so the two terms in the big parentheses are equal only when $\eta=1/2$. As the expected profit is linear in η, the optimal η must be 0 or 1 or 1/2. However, if $\eta=0$, then the first term is larger, meaning that a leading-1 brand has more consumer visits, so the optimal η should be 1. Similarly for $\eta=1$. Therefore, only the pooling equilibrium ($\eta=1/2$) can exist.

The second possibility is that the two-product firm sets prices to be $((1+v)/2,v)$ or $(v,(1+v)/2)$ for its brand. Suppose the two-product firm chooses prices $((1+v)/2,v)$. Keep the notion for β^i and $\tilde{p}_i$. Type-L consumers' expected surplus of visiting a leading-1 brand is$\beta^1(v-\tilde{p}_1)+(1-\beta^1)\frac{(1-v)^2}{2}$. Type-$R$ consumers' expected surplus of visiting a leading-1 brand is $\beta^1\frac{(1-\tilde{p}_1)^2}{2}+(1-\beta^1)\frac{(1-v)^2}{8}$. j_1 firm maximizes profit from two types of consumers by choosing $\tilde{p}_1$ that satisfies

$$\tilde{p}_1=\begin{cases}\frac{1}{2}+\frac{\beta^1(v-\tilde{p}_1)+(1-\beta^1)(1-v)^2/2}{\beta^1(1-\tilde{p}_1)^2+(1-\beta^1)(1-v)^2/4} & \text{if } \beta^1>\frac{5-2v}{1+6v},\\ v & \text{if } \beta^1\le\frac{5-2v}{1+6v}.\end{cases}$$

It is easy to show that $\tilde{p}_1$ is decreasing in β^1. Similarly we can get the expression for $\tilde{p}_2$. For the two-product firm, the expected profit of mixing with probability η is

$$\frac{\eta}{2\bar{c}}\left(\left(\beta^1(v-\tilde{p}_1)+(1-\beta^1)\frac{(1-v)^2}{2}\right)v+\left(\beta^1\frac{(1-\tilde{p}_1)^2}{2}+(1-\beta^1)\frac{(1-v)^2}{8}\right)\frac{(1+v)^2}{4}\right)$$
$$+\frac{1-\eta}{2\bar{c}}\left(\left(\beta^2\frac{(1-\tilde{p}_2)^2}{2}+(1-\beta^2)\frac{(1-v)^2}{2}\right)v+\left(\beta^2(v-\tilde{p}_2)+(1-\beta^2)\frac{(1-v)^2}{8}\right)\frac{(1+v)^2}{4}\right).$$

In general the two terms in the big parentheses are equal only when $\eta=\tilde{\eta}$ for some $\tilde{\eta}>1/2$. Therefore, the optimal η must be 0 or 1 or $\tilde{\eta}$. However, if $\eta=\tilde{\eta}$, the two-product firm can deviate to $(v,(1+v)/2)$ for higher profit. In addition, it's better to deviate to $(v,(1+v)/2)$ if $\eta=0$ as more type-L consumers would visit. If $\eta=1$, it's better for the two-product firm to deviate to $(v,(1+v)/2)$ and $\eta=0$, i.e. creating a leading-2 brand instead of a leading-1 brand. Therefore, there is no single-brand equilibrium with asymmetric prices. □

Proof of Proposition 1. We first show that the separating case is an equilibrium. If the deviating two-product firm creates just one brand and sets $p_1=p_2=v$ for its sole brand, it earns

$$\frac{1}{2\bar{c}}\left(v-p^*+\frac{(1-p^*)^2}{2}\right)v<\frac{v(1-v)^2}{2\bar{c}}.$$

To see this, as p^* is given by equation (1), we can get the first order derivative as $\frac{\partial p^*}{\partial v}=\frac{1}{3{p^*}^2-5p^*+3}>$

0. Therefore,

$$\frac{\partial}{\partial v}\left(v - p^* + \frac{(1-p^*)^2}{2} - (1-v)^2\right) = \frac{\left(2 - (2p^*-1)(2-2p^*+p^{*^2})\right)(3p^{*^2} - 5p^* + 3) + (2-p^*)}{3p^{*^2} - 5p^* + 3}$$

As $p^* \in (0.5, v) \subset (0.5, 1)$, the denominator is always positive and the numerator is strictly decreasing in p^* and always positive, so the difference in the profits is strictly decreasing. At $v = 1$, the two profits are equal. Therefore, the initial inequality holds. Thus, the two-product firm has no incentive to deviate and the separating case is indeed an equilibrium.

Next we show that the pooling case is not an equilibrium. In the pooling case, the two-product firm's expected profit is

$$\frac{1}{2\bar{c}}\left(\gamma\left(v - p^P + \frac{(1-p^P)^2}{2}\right) + (1-\gamma)(1-v)^2\right)v.$$

If the two-product firm deviates by establishing two brands with distinct leading products, consumers would know both products are available, and the firm's optimal profit, as in the separating equilibrium, is $\frac{v(1-v)^2}{2\bar{c}}$. The deviating firm would earn a higher profit as

$$(1-v)^2 > v - p^* + \frac{(1-p^*)^2}{2} > v - p^P + \frac{(1-p^P)^2}{2},$$

Therefore, the pooling equilibrium is not viable. This result can then be extended to any single-brand case as we already show in Lemma 2. In conclusion, there is an unique equilibrium when the platform presents all brands to all consumers: the separating equilibrium. □

Proof of Proposition 2. The profit of the two-product firm under the two-brand equilibrium is higher because

$$\frac{v}{2\bar{c}}\left(\gamma\frac{(1-p^P)^2}{2} + \gamma(v - p^P) + (1-\gamma)(1-v)^2\right) < \frac{v(1-v)^2}{2\bar{c}} \Leftrightarrow v - p^P + \frac{(1-p^P)^2}{2} < (1-v)^2,$$

which always holds. Consumers' expected surplus under the two-brand equilibrium is higher if

$$\left(p^* - p^P\right)\left(2 - \frac{p^* + p^P}{2}\right) < 0,$$

which always holds. Therefore, consumers also benefit from the presence of virtual brands. Lastly, for the single-product firm, the change in expected profit from two-brand equilibrium to single-brand equilibrium is

$$\frac{p^{P^2}(\gamma(1-p^P)^2 + (1-\gamma)(1-v)^2)}{4\bar{c}} - \frac{p^{*^2}(1-p^*)^2}{4\bar{c}} < \frac{p^{P^2}(1-p^P)^2}{4\bar{c}} - \frac{p^{*^2}(1-p^*)^2}{4\bar{c}} < 0.$$

Therefore, the single-product firm also benefits from the presence of virtual brands. □

Proof of Proposition 3. In the privacy environment, if the platform selectively recommends a brand not contingent on consumer types, as we argued, the equilibrium outcome will be the same as one of the pooling equilibria where the platform presents all available brands. We have shown that consumers and all firm types are better off under full recommendation, so is the platform.

Now consider targeted recommendation in no-privacy environment. Suppose for now that the j_{12} firm charges price (v, v) for both brands. First consider the platform recommends each consumer the brand leading with her familiar product if there are two distinct brands. Denote the equilibrium price charged by the single-product firm as p^F. The expected profit for the j_1 firm of charging price p is

$$\frac{1}{2\bar{c}}\left(\frac{\gamma}{2-\gamma}(v-p^F)+\frac{2-2\gamma}{2-\gamma}\frac{(1-v)^2}{2}\right)p+\frac{1}{2\bar{c}}\frac{(1-p^F)^2}{2}p(1-p),$$

so the optimal price is given by equation (4). Compared with the separating equilibrium, j_1 firm's profit is lower if

$$\frac{{p^F}^2(1-p^F)^2}{4\bar{c}}<\frac{p^*(1-p^*)^2}{4\bar{c}}\Leftarrow p^F>p^*>\frac{1}{2},$$

which always holds, and j_{12} firm's profit is lower if

$$\frac{\gamma}{2-\gamma}(v-p^F)+\frac{2-2\gamma}{2-\gamma}\frac{(1-v)^2}{2}<\frac{(1-v)^2}{2}\iff v-p^F<\frac{(1-v)^2}{2},$$

which always holds. Therefore, the platform would earn a lower expected revenue by adopting this strategy than full recommendation.

Next consider the platform recommends each consumer the brand leading with her unfamiliar product if there are two distinct brands. The expected profit for the j_1 firm of charging price p is

$$\frac{1}{2\bar{c}}(v-p^U)p+\frac{1}{2\bar{c}}\left(\frac{\gamma}{2-\gamma}\frac{(1-p^U)^2}{2}+\frac{2-2\gamma}{2-\gamma}\frac{(1-v)^2}{2}\right)p(1-p),$$

so the optimal price p^U is given by equation (5). Taking the first order derivative with respect to γ, we have

$$\frac{\partial p^U}{\partial\gamma}=\frac{\frac{v-p^U}{2p^U-1}+\frac{(1-p^U)^2}{2}-(1-v)^2}{\gamma(1-p^U)+(2-\gamma)\frac{1-2v}{(2p^U-1)^2}}=\frac{1}{2-\gamma}\frac{(1-p^U)^2-(1-v)^2}{\gamma(1-p^U)+(2-\gamma)\frac{1-2v}{(2p^U-1)^2}}.$$

The numerator is obviously positive. For the denominator, the partial derivative with respect to p^U equals

$$-\gamma-(2-\gamma)\frac{4(1-2v)}{(2p^U-1)^3}>-\gamma+(2-\gamma)\frac{4}{(2v-1)^2}>0.$$

Therefore, the denominator

$$\gamma(1-p^U)+(2-\gamma)\frac{1-2v}{(2p^U-1)^2}<\gamma(1-v)-\frac{2-\gamma}{2v-1}<0.$$

This means that p^U is decreasing in γ. Taking the second order derivative with respect to γ, we have

$$\frac{\partial^2 p^U}{\partial \gamma^2} = \frac{\left((2-\gamma)\frac{4(1-2v)}{(2p^U-1)^3}+\gamma\right)\left(\frac{\partial p^U}{\partial \gamma}\right)^2 + 2\left(\frac{1-2v}{(2p^U-1)^2}-(1-p^U)\right)\frac{\partial p^U}{\partial \gamma}}{\gamma(1-p^U)+(2-\gamma)\frac{1-2v}{(2p^U-1)^2}}.$$

Compared with the separating equilibrium, j_1 firm's profit is lower if

$$\frac{p^{U2}}{2\bar{c}}\left(\frac{\gamma}{2-\gamma}\frac{(1-p^U)^2}{2}+\frac{2-2\gamma}{2-\gamma}\frac{(1-v)^2}{2}\right) < \frac{p^*(1-p^*)^2}{4\bar{c}} \Leftarrow p^U > p^* > \frac{1}{2},$$

which always holds, and the two-product firm's profit is higher if

$$\frac{\gamma}{2-\gamma}\frac{(1-p^U)^2}{2}+\frac{2-2\gamma}{2-\gamma}\frac{(1-v)^2}{2} > \frac{(1-v)^2}{2} \Leftarrow p^U > p^* > \frac{1}{2},$$

which always holds. For the platform, under separating equilibrium, the industry profit is expression (3); under the targeted recommendation of unfamiliar product, the industry profit is

$$\frac{1}{\bar{c}}\left(\frac{\gamma}{2-\gamma}\frac{(1-p^U)^2}{2}+\frac{2-2\gamma}{2-\gamma}\frac{(1-v)^2}{2}\right)\left(\gamma\frac{p^{U2}}{2}+(1-\gamma)v\right). \tag{7}$$

It is easy to see that the above two expected profits are the same when $\gamma \in \{0,1\}$. The partial derivative of the industry profit (3) in separating equilibrium with respect to γ is constant at

$$\frac{p^{*2}(1-p^*)^2}{4\bar{c}} - \frac{(1-v)^2}{2\bar{c}}v < 0.$$

The first order partial derivative of the industry profit (7) in targeted recommendation with respect to γ is

$$\frac{1}{\bar{c}}\left(\frac{p^{U2}}{2}-v\right)\frac{v-p^U}{2p^U-1}+\frac{1}{\bar{c}}\left(\left(\gamma\frac{p^{U2}}{2}+(1-\gamma)v\right)\frac{1-2v}{(2p^U-1)^2}+\gamma p^U\frac{v-p^U}{2p^U-1}\right)\frac{\partial p^U}{\partial \gamma}.$$

The second order partial derivative of the targeted industry profit with respect to γ is

$$\frac{1}{\bar{c}}\frac{A_1\frac{\partial p^U}{\partial \gamma}+A_2\left(\frac{\partial p^U}{\partial \gamma}\right)^2}{\gamma(1-p^U)+(2-\gamma)\frac{1-2v}{(2p^U-1)^2}},$$

where the denominator has been shown to be negative, and

$$\begin{aligned} A_1 =& \frac{2(2v-1)}{(2p^U-1)^4}\left((1-v)p^U(2p^U-1)^2-(v-p^U)(2p^U-1)-(v-p^{U^2})(2v-1)\right) \\ <& \frac{2(2v-1)}{(2p^U-1)^3}\left((1-v)p^U(2p^U-1)-2v+p^U+p^{U^2}\right) < 0, \end{aligned}$$

$$A_2 = \frac{\gamma(2v-1)}{(2p^U-1)^3}\left((2-\gamma)(3p^U - v) - (2p^U-1)\left(2\gamma p^U - \frac{3}{2}\gamma p^{U^2} + (1-\gamma)v\right)\right.$$
$$\left. + 4(1-p^U)\left(\gamma\frac{p^{U^2}}{2} + (1-\gamma)v\right)\right) + \gamma^2\frac{v-p^U}{2p^U-1} > 0.$$

The signs of A_1, A_2 can be shown by first showing the terms in the big parentheses are increasing in p^U. Hence $A_1\frac{\partial p^U}{\partial\gamma} + A_2\left(\frac{\partial p^U}{\partial\gamma}\right)^2 > 0$. Therefore the first order partial derivative of the selective recommendation profit is strictly decreasing in γ while the first order partial derivative of the separating equilibrium profit is constant, so the difference between the targeted industry profit and the separating industry profit must be first increasing and then decreasing in γ. Hence the difference is always above 0 as the differences at the two end points are 0.

The next question to consider is whether the two-product firm would indeed choose prices (v, v). With targeted recommendation of the two-product brands, the j_{12} firm can charge asymmetric prices, $(1+v)/2$ for the leading product and v for the second product to maximize profit. The expected profit for the j_1 firm of charging price p is

$$\frac{1}{2\bar{c}}(v - p^{U'})p + \frac{1}{2\bar{c}}\left(\frac{\gamma}{2-\gamma}\frac{(1-p^{U'})^2}{2} + \frac{2-2\gamma}{2-\gamma}\frac{(1-v)^2}{8}\right)p(1-p),$$

so the optimal price $p^{U'}$ is given by

$$p^{U'} = \frac{1}{2} + \frac{v - p^{U'}}{\frac{\gamma}{2-\gamma}(1-p^{U'})^2 + \frac{2-2\gamma}{2-\gamma}\frac{(1-v)^2}{4}} \in (p^U, v). \tag{8}$$

When $\gamma = 1$, $p^{U'} = p^U = p_1^*$. Taking the first order derivative with respect to γ, we have

$$\frac{\partial p^{U'}}{\partial\gamma} = \frac{\frac{v-p^{U'}}{2p^{U'}-1} + \frac{(1-p^{U'})^2}{2} - \frac{(1-v)^2}{4}}{\gamma(1-p^{U'}) + (2-\gamma)\frac{1-2v}{(2p^{U'}-1)^2}} = \frac{1}{2-\gamma}\frac{(1-p^{U'})^2 - \frac{(1-v)^2}{4}}{\gamma(1-p^{U'}) + (2-\gamma)\frac{1-2v}{(2p^{U'}-1)^2}}.$$

The numerator is obviously positive. The denominator, as before, can be shown to be negative. This means that p^U is decreasing in γ. Taking the second order derivative with respect to γ, we have

$$\frac{\partial^2 p^{U'}}{\partial\gamma^2} = \frac{\left((2-\gamma)\frac{4(1-2v)}{(2p^{U'}-1)^3} + \gamma\right)\left(\frac{\partial p^{U'}}{\partial\gamma}\right)^2 + 2\left(\frac{1-2v}{(2p^{U'}-1)^2} - (1-p^{U'})\right)\frac{\partial p^{U'}}{\partial\gamma}}{\gamma(1-p^{U'}) + (2-\gamma)\frac{1-2v}{(2p^{U'}-1)^2}}.$$

For the platform, under separating equilibrium, the industry profit is expression (3); under the targeted recommendation of unfamiliar product, the industry profit is

$$\frac{1}{\bar{c}}\left(\frac{\gamma}{2-\gamma}\frac{(1-p^{U'})^2}{2} + \frac{2-2\gamma}{2-\gamma}\frac{(1-v)^2}{8}\right)\left(\gamma\frac{p_1^{U'2}}{2} + (1-\gamma)\frac{(1+v)^2}{4}\right). \tag{9}$$

It is easy to see that the expected profit under targeted recommendation is the same as the profit under the separating equilibrium when $\gamma = 0$, and is lower when $\gamma = 1$. The first order partial derivative of the industry profit (9) in targeted recommendation with respect to γ is

$$\frac{1}{\bar{c}}\left(\frac{p_1^{U'2}}{2} - \frac{(1+v)^2}{4}\right)\frac{v-p^{U'}}{2p^{U'}-1}+\frac{1}{\bar{c}}\left(\left(\gamma\frac{p_1^{U'2}}{2}+(1-\gamma)\frac{(1+v)^2}{4}\right)\frac{1-2v}{(2p^{U'}-1)^2}+\gamma p^{U'}\frac{v-p^U}{2p^{U'}-1}\right)\frac{\partial p^{U'}}{\partial \gamma}.$$

The second order partial derivative of the targeted industry profit with respect to γ is

$$\frac{1}{\bar{c}}\frac{A_1'\frac{\partial p^{U'}}{\partial\gamma} + A_2'\left(\frac{\partial p^{U'}}{\partial\gamma}\right)^2}{\gamma(1-p^{U'}) + (2-\gamma)\frac{1-2v}{(2p^{U'}-1)^2}},$$

where the denominator has been shown to be negative, and similar to above, it can be shown that $A_1' < 0$ and $A_2' > 0$. Hence $A_1\frac{\partial p^U}{\partial\gamma} + A_2\left(\frac{\partial p^U}{\partial\gamma}\right)^2 > 0$. Therefore the first order partial derivative of the selective recommendation profit is strictly decreasing in γ while the first order partial derivative of the separating equilibrium profit is constant. In addition, the difference in the first order derivatives at $\gamma = 0$ is negative. Therefore, the difference in the first order derivatives is always below 0 and the platform earns a lower profit than the profit under separating equilibrium.

Combining the above analyses, the platform would earn the highest profit when it targetedly recommends the two-product brand leading with consumers' unfamiliar product. However, the two-product firm would deviate to asymmetric pricing if the platform always targetedly recommends this way. The platform can sustain the prices (v, v) by targetedly recommending the two-product brand leading with consumers' unfamiliar product with probability ϕ and recommending both brands with probability $1-\phi$ when two distinct brands are available. The optimal ϕ satisfies

$$\frac{(1-\phi)\frac{(1-v)^2}{4}}{\phi\left(\frac{\gamma}{\gamma+2(1-\gamma)\phi}\frac{(1-p_1^{U*})^2}{2} + \frac{2(1-\gamma)\phi}{\gamma+2(1-\gamma)\phi}\frac{(1-v)^2}{2}\right) + (1-\phi)\frac{(1-v)^2}{4}} = \frac{(1-v)^2}{4v^2} \tag{10}$$

where p_1^{U*} solves

$$p_1^{U*} = \frac{1}{2} + \frac{v - p_1^{U*}}{\frac{\gamma}{\gamma+2(1-\gamma)\phi}\frac{(1-p_1^{U*})^2}{2} + \frac{2(1-\gamma)\phi}{\gamma+2(1-\gamma)\phi}\frac{(1-v)^2}{2}} \in (p^P, p^U). \tag{11}$$

At last, we show an example in which the platform finds it optimal to targetedly recommend to each consumer the brand leading with their familiar product. Consider $v = 0.9$ and $\gamma = 0.1$. The j_{12} firm would optimally choose asymmetric prices, v for the familiar product and $(1+v)/2$ for the unfamiliar product. Denote the equilibrium price charged by the single-product firm as $p^{F'}$. The expected profit for the j_1 firm of charging price p is

$$\frac{1}{2\bar{c}}\left(\frac{\gamma}{2-\gamma}(v-p^{F'}) + \frac{2-2\gamma}{2-\gamma}\frac{(1-v)^2}{8}\right)p + \frac{1}{2\bar{c}}\frac{(1-p^{F'})^2}{2}p(1-p),$$

and the optimal price can be solved as $p^{F'} \approx 0.608$. For the platform, the industry profit is given by

$$\gamma\frac{p^{F'2}(1-p^{F'})^2}{4\bar{c}} + (1-\gamma)\frac{1}{\bar{c}}\left(\frac{\gamma}{2-\gamma}(v-p^{F'}) + \frac{2-2\gamma}{2-\gamma}\frac{(1-v)^2}{2}\right)v \approx \frac{0.0177}{\bar{c}}.$$

When the platform chooses full recommendation, the equilibrium price charged by the single-product firm is $p^* \approx 0.896$, and the industry profit is given by

$$\gamma\frac{{p^*}^2(1-p^*)^2}{4\bar{c}} + (1-\gamma)\frac{(1-v)^2}{2\bar{c}}v \approx \frac{0.0177}{\bar{c}} \approx \frac{0.00427}{\bar{c}}.$$

When the platform chooses optimal targeted recommendation of two-product brand leading with unfamiliar product, the equilibrium price charged by the single-product firm is $\phi \approx 0.994, p^{U*} \approx 0.896$, and the industry profit is approximately $0.00427/\bar{c}$. Therefore, the targeted recommendation of the two-product brand leading with consumers' familiar product is the optimal strategy by the platform.

□

Proof of Proposition 4. If the two-product firm creates two brands with distinct products, the optimal prices for both products are given by (6). We have $\frac{\partial\hat{p}}{\partial v} = \frac{2\hat{p}}{2(1-v)+\hat{p}+3\hat{p}^2} > 0$. As the search order requires $\hat{p} > \sqrt{2v-1}$, we have $2v - 3\hat{p} < 2v - 3\sqrt{2v-1} < 0$. Then we can get the conditions $v < 2-\sqrt{2}$ and $\hat{p} \in (2v/3, \sqrt{2}-1)$ for the equilibrium to hold. The maximized profit for the two-product firm is $\frac{\hat{p}((1-\hat{p})^2+2\hat{p}^2)}{4\bar{c}}$.

First we compare with the separating equilibrium under full recommendation. As the single-product firm earns the same profit as in separating equilibrium without ban, the platform earns a higher profit than the separating equilibrium case without ban if the two-product firm also earns a higher profit, i.e.

$$\frac{\hat{p}((1-\hat{p})^2+2\hat{p}^2)}{4\bar{c}} > \frac{v(1-v)^2}{2\bar{c}}.$$

As the left hand side is increasing in $\hat{p}$ (and hence increasing in v) whereas the right hand side is strictly decreasing in v, it is sufficient to show that the inequality holds when $v = 1/2$. When $v = 1/2$, $\hat{p} \approx 0.376086$, the left hand side equals $0.0632/\bar{c}$, and the right hand side equals $0.0625/\bar{c}$, which is smaller. By induction, the two-product firm earns a higher profit than creating a single brand. Therefore, it is optimal for the two-product firm to create such two distinct brands when the platform presents all available brands to consumers.

Next we compare with the targeted recommendation of unfamiliar product brand without ban. With the ban, the industry profit is

$$\gamma\frac{{p^*}^2(1-p^*)^2}{4\bar{c}} + (1-\gamma)\frac{\hat{p}((1-\hat{p})^2+2\hat{p}^2)}{4\bar{c}}.$$

The first order derivative with respect to γ is constant at

$$\frac{{p^*}^2(1-p^*)^2}{4\bar{c}} - \frac{\hat{p}((1-\hat{p})^2+2\hat{p}^2)}{4\bar{c}} < 0.$$

Under the targeted recommendation of unfamiliar product, the industry profit is

$$\frac{1}{\bar{c}}\left(\frac{\gamma}{2-\gamma}\frac{(1-p^U)^2}{2}+\frac{2-2\gamma}{2-\gamma}\frac{(1-v)^2}{2}\right)\left(\gamma\frac{p^{U2}}{2}+(1-\gamma)v\right).$$

We've already shown in the previous proofs that the first order derivative with respect to γ is strictly decreasing. Taking $\gamma \to 1$, we have $p^U \to p^*$, and the first order derivative of the difference between industry profit under targeted recommendation of unfamiliar product and industry profit under ban is

$$>\left(-\frac{v(1-p^U)^2}{2\bar{c}}+\frac{\hat{p}((1-\hat{p})^2+2\hat{p}^2)}{4\bar{c}}\right)+\left(\frac{p^{U2}}{2\bar{c}}\frac{1-2v}{(2p^U-1)^2}+p^U\frac{(1-p^U)^2}{2\bar{c}}\right)\frac{(1-p^U)^2-(1-v)^2}{(1-p^U)+\frac{1-2v}{(2p^U-1)^2}}$$

For the first part, $-\frac{v(1-p^U)^2}{2}$ is increasing in v with infimum at -0.0625, and $\frac{\hat{p}((1-\hat{p})^2+2\hat{p}^2)}{4\bar{c}}$ is strictly increasing in v with infimum at 0.0632, so the first part is positive. As we have shown in $\partial p^U/\partial\gamma$ that

$$(1-p^U)+\frac{1-2v}{(2p^U-1)^2}<0,$$

we have

$$\frac{p^{U2}}{2\bar{c}}\frac{1-2v}{(2p^U-1)^2}+p^U\frac{(1-p^U)^2}{2\bar{c}}<-\frac{p^{U2}}{2\bar{c}}(1-p^U)+p^U\frac{(1-p^U)^2}{2\bar{c}}=p^U\frac{(1-p^U)(1-2p^U)}{2\bar{c}}<0.$$

Therefore, the second part is also positive, so the difference is always increasing in γ. As the difference is negative at $\gamma=0$ and 0 at $\gamma=1$, the difference is negative for $\gamma\in(0,1)$. As the targeted recommendation of unfamiliar product brand with optimal mixing ϕ^* yields a even lower industry profit, the platform would sustain such an equilibrium instead of targeted recommendation.

Lastly, we compare the profit of the two-product firm in these two cases. The difference between the profit under targeted recommendation without ban and the profit under ban is

$$\frac{1}{\bar{c}}\left(\frac{\gamma}{2-\gamma}\frac{(1-p^U)^2}{2}+\frac{2-2\gamma}{2-\gamma}\frac{(1-v)^2}{2}\right)v-\frac{\hat{p}((1-\hat{p})^2+2\hat{p}^2)}{4\bar{c}}<\frac{(1-p_1^*)^2}{2\bar{c}}v-\frac{\hat{p}((1-\hat{p})^2+2\hat{p}^2)}{4\bar{c}}.$$

As the first profit is decreasing in v and the second profit is increasing in v, we have the difference is smaller than the value at $v=1/2$, which is

$$\frac{0.0625}{\bar{c}}-\frac{0.0632}{\bar{c}}<0.$$

Therefore, the two-product firm earns higher profit under ban than under the targeted recommendation without ban. □

Proof of Proposition 5. We consider the three cases of selective recommendation here. For the first case, the two-product firm would set prices to be v for both products. Let the price set by the single-product firm be p'. The expected consumer surplus of visiting her familiar product brand is $\frac{\gamma}{2-\gamma}(v-p')$, and the expected single-product firm profit of charging price p is $\frac{1}{2}\frac{\gamma(v-p')}{(2-\gamma)\bar{c}}p + \frac{1}{2}\frac{(1-p')^2}{2\bar{c}}p(1-p)$, so the optimal price is given by

$$p' = \frac{1}{2} + \frac{\gamma}{2-\gamma}\frac{v-p'}{(1-p')^2} \in (\frac{1}{2}, p^*).$$

The maximized profit of the single-product firm is $\frac{1}{4\bar{c}}p'^2(1-p')^2$, and the maximized profit of the two-product firm is $\frac{\gamma}{2-\gamma}\frac{v-p'}{\bar{c}}v$. For the platform, the industry profit is

$$\gamma\frac{1}{4\bar{c}}p'^2(1-p')^2 + (1-\gamma)\frac{1}{\bar{c}}\frac{\gamma}{2-\gamma}(v-p')v.$$

Alternatively, if the firm deviates to just create one brand with both products and randomly picks the leading product with equal probability, it will pool with the single-product firm offering the same leading product brand. The deviating profit is

$$\frac{1}{2}\frac{\gamma(v-p')}{(2-\gamma)\bar{c}}v + \frac{1}{2}\frac{(1-p')^2}{2\bar{c}}v > \frac{\gamma}{2-\gamma}\frac{v-p'}{\bar{c}}v.$$

Therefore, this cannot be an equilibrium.

For the second case, the two-product firm would set prices to be 1/2 for both products. Let the single-product firm would set price to be p''. The expected consumer surplus of visiting her unfamiliar product brand is

$$\frac{\gamma}{2-\gamma}\frac{(1-p'')^2}{2} + \frac{2-2\gamma}{2-\gamma}\frac{1}{8},$$

and the expected single-product firm profit of charging price p is

$$\frac{1}{2}\frac{v-p''}{\bar{c}}p + \frac{1}{2\bar{c}}\left(\frac{\gamma}{2-\gamma}\frac{(1-p'')^2}{2} + \frac{2-2\gamma}{2-\gamma}\frac{1}{8}\right)p(1-p),$$

so the optimal price is given by

$$p'' = \frac{1}{2} + \frac{v-p''}{\frac{\gamma}{2-\gamma}(1-p'')^2 + \frac{2-2\gamma}{2-\gamma}\frac{1}{4}} \in (p^*, v).$$

The maximized profit of the two-product firm is

$$\left(\frac{\gamma}{2-\gamma}\frac{(1-p'')^2}{2} + \frac{2-2\gamma}{2-\gamma}\frac{1}{8}\right)\frac{1}{4\bar{c}}.$$

Alternatively, if the firm deviates to just create one brand with both products and randomly picks the leading product with equal probability, it will pool with the single-product firm offering the same leading product brand. The deviating profit is

$$\frac{1}{2}\frac{v-p''}{\bar{c}}v+\frac{1}{2\bar{c}}\left(\frac{\gamma}{2-\gamma}\frac{(1-p'')^2}{2}+\frac{2-2\gamma}{2-\gamma}\frac{1}{8}\right)v>\left(\frac{\gamma}{2-\gamma}\frac{(1-p'')^2}{2}+\frac{2-2\gamma}{2-\gamma}\frac{1}{8}\right)\frac{1}{4\bar{c}}.$$

Therefore, this cannot be an equilibrium.

For the third case, the platform randomly shows consumers one brand when there are two distinct brands. Each consumer is equally likely to see either type of brands, and each brand has equal chance to be shown to either type of consumers. All firms would just set product price to be p^*. The expected consumer surplus conditional on visit is $v - p^*$ for a familiar product brand and $(1-p^*)^2/2$ for an unfamiliar product brand. The expected profit of the both single-product and two-product firm is $\frac{{p^*}^2(1-p^*)^2}{2\bar{c}}$. If the two-product firm deviates to create one brand with both-products, the expected profit is

$$\frac{1}{2\bar{c}}\left(v-p^*+\frac{(1-p^*)^2}{2}\right)v=\frac{p^*v(1-p^*)^2}{2\bar{c}}>\frac{{p^*}^2(1-p^*)^2}{2\bar{c}}.$$

Therefore, this cannot be an equilibrium. When $v > 2-\sqrt{2}$, there is no equilibrium where the two-product firm creates two distinct brands. If it just creates a single brand, we've already shown in Lemma 2 that the pooling equilibrium is the unique equilibrium. □